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PICTORIAL RADIO

BY

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Part I [2]

INTRODUCTION

"Pictorial Radio" is an expression coined to provide a short, descriptive title. Perhaps a more accurate, but far less handy, title might be "Multi-Coordinate or Graphic Indicating Radio Systems for Obtaining Bi- or Tri-Dimensional Information Transmitted or Reflected from a Plurality of Geometrically Related Points." Particular attention will be given to radio vision systems, other than television, adapted to display the received information in graphic form along two or more coordinates. By way of example, one coordinate might be distance; the other, azimuth. Or the coordinates might be azimuth and elevation, or width and depth. Tri-Dimensional representation is not excluded. One practical example is Plan Position Indicating Radar—better known as PPI.

THE PIONEER PERIOD 1887-1900

We are probably too close to the latest events to write an accurate and complete history of multi-coordinate radio systems but it is not too early to begin collecting the evidence. The scope of the evidence might be indicated by an analogous art such as photography. In photography the essentials would be: (1) electromagnetic waves (for example, light), (2) a lens or pinhole to establish an image, and (3) a wave responsive medium (such as ground glass plate for viewing the image or a photosensitive film for recording the image). The corresponding radio

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[2] Part II will appear in this JOURNAL for February, 1952.

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elements are: (1) radio frequency waves (preferably of very short wave length), (2) a lens or the equivalent, (3) a wave detector or responder (such as a resonant circuit), and (4) a visual signal indicator. The last two elements might be combined as in the Hertz resonator with its signal indicating spark gap (1).[3] Hertz described all the elements. He may have painstakingly traced the radio fields from the feeble source of electromagnetic radiations that were refracted, reflected, concentrated, and converted into visible light in his small laboratory about the year 1887. In a sense, he may have had the first "two-coordinate" system, but it is unlikely that Hertz or observers in his laboratory had before them a pictorial radio representation of the type developed in the first half of the present century.

Before leaving the pioneer period let us consider briefly some of the work of Fitzgerald, Lodge, and Marconi.

Fitzgerald and Trouton

In the fall of 1888 Professor Fitzgerald and Fred T. Trouton repeated a number of Hertz's experiments (2). They first used a dipole

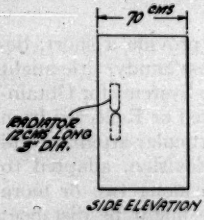

FIG. 1.

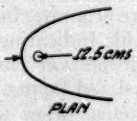

FIG. 2.

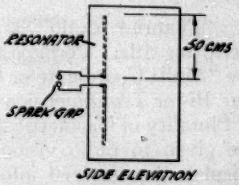

FIG. 3.

with end plates and brass spark knobs as the radiator or vibrator and a circular loop terminating in a spark gap as the resonator. While electric waves were reflected from conducting surfaces, the experimenters were also trying to obtain reflections from non-conductors.

The experiments with non-conductors were not satisfactory and, to obviate a difficulty, the experimenters decided to try Hertz's cylindrical parabolic mirrors for concentrating the radiation. The reflectors were made of a zinc sheet as shown in Figs. 1, 2, and 3.

The radiator, consisting of two brass cylinders, each 12 cm. long and 3 cm. in diameter with rounded sparking ends, was placed in the focal line. The focal line was 12.5 cm. from the vertex of the mirrors. The mirrors were 70 cm. deep. The receiving mirror included a resonator formed with two thick wires each 50 cm. long. The inner ends of the two wires were connected to thin wires that passed through holes in the mirror to a spark gap at the rear of the mirror as shown in Fig. 3.

[3] The boldface numbers in parentheses refer to the references to Part I of this paper.

When the radiator was energized by an induction coil, a concentrated beam of electric waves was formed. While Fitzgerald and Trouton did not obtain reflections from glass plates, they did observe sparks that were detected in the resonator-mirror receiver after the waves were reflected from a wall 3 ft. thick. The use of Hertzian waves for experimental purposes was suggested, but no suggestion of communication, and much less "pictorial radio," was found in their report.

Lodge and Howard

Professor Oliver Lodge and Dr. James L. Howard presented a paper before the Physical Society on May 11, 1889 (3). The paper, entitled "On Electric Radiation and its Concentration by Lenses," described theoretical and experimental work involving electric wave transmission and reception. The point of present interest is the use of lenses to concentrate the waves.

The oscillator or source of radiation consisted of a dipole including a spark gap formed by a pair of brass spheres. Each sphere was 2 cm. in diameter. The spheres were mounted on the ends of rods 6 cm. long.

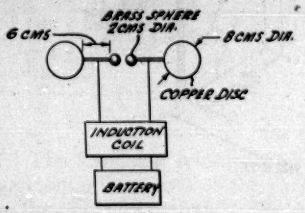

FIG. 4.

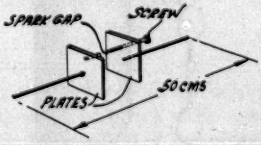

FIG. 5.

Thin copper discs 8 cm. in diameter were fastened on the free ends of the rods. The power was supplied by an induction coil, operated by a storage battery (Fig. 4). The operating wave length was 100 cm.

The resonator, or receiver and detector, was formed of two pieces of copper wire (No. 13 B.W.G.) each 25 cm. long. The inner ends of the rods terminated in small rectangular brass plates. Each plate carried a point; one was adjustable by a screw so that the distance between the points could be varied (Fig. 5).

Two lenses were made of common mineral pitch. Each lens was cast in the form of a hyperbolic cylinder, having a plane surface 85 cm. high and 90 cm. broad. The greatest thickness (from vertex of hyperbola to plane surface of lens) was 21 cm. Each lens weighed 3 cwt. A single lens was designed to converge a bundle of parallel rays falling normally on its plane surface to a line of foci coinciding with the outer foci of its principal hyperbolic sections. Conversely, rays from the focal line, falling on the curved surface, are converted into a bundle of parallel rays emerging normal to the plane surface.

The lenses were placed 180 cm. apart with their plane surfaces parallel as shown in Fig. 6. The oscillator was arranged 51 cm. from the vertex of the nearer lens. Due to a slight miscalculation in the index of refraction, the oscillator was only approximately at the focal line. The plane of the oscillator discs was parallel to the plane of the lens for the maximum radiation through the lenses.

The direct radiations—unaided by the lens—were detected by the resonator at a maximum distance of 120 cm. This required clean spheres on the oscillator, and a very dark room to detect sparking at the resonator points. Concentration of the radiation by the first lens was very well marked. It was noted that the radiations were almost as intense in the axial plane after passing through the maximum lens thickness as when impinging on its curved surface.

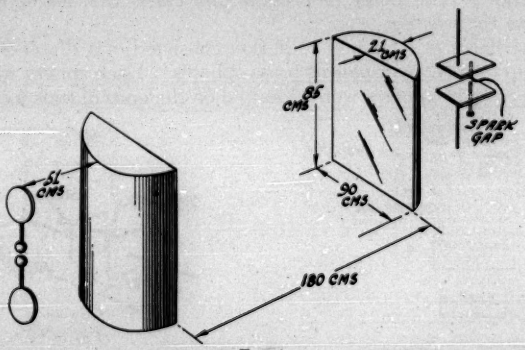

FIG. 6.

The resonator gave sparks of fair intensity at the focal line of the second lens in responding to radiations passing through both lenses. The maximum distance, at which any traces of sparking were observed, was 450 cm. from the oscillator, against 120 cm. without the aid of the lenses. An exploration of the region beyond the second lens demonstrated clearly the converging or focusing effect.

While the space between the lenses was explored to determine the character of the field, no attempt was made to increase the range by increasing the distance between the lenses. Apparently Lodge and Howard were content "to confirm the identity of electric radiation and light" and most modestly attributed to their experiments "merely a slight extension of the famous researches of Hertz."

Marconi

About 1896 Senator Guglielmo Marconi (4) used cylindrical parabolic reflectors at the transmitter and at the receiver. In the case of

the transmitter the spark discharge balls and associated radiator elements were placed in the focal line of the reflector (Fig. 7). At the receiver the wave energy was collected by two metal rods or pieces that were positioned at the focus of the reflector and were connected to the wave detector or coherer (Fig. 8). The primary purpose of the reflectors was to increase the effective communication range, for that was Marconi's great contribution at the time when most of the other pioneers were more concerned with demonstrating that the radio frequency waves obeyed the laws of transmission of optical waves. Marconi soon discovered, however, that his elevated antennas increased the range more than the reflectors. Apparently he abandoned the use of reflectors for many years but returned to them about 1916 in his later experiments with short waves. Marconi was aware of the improvement in receiver selectivity due to the directivity of the beam (5).

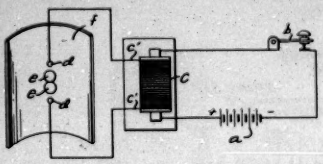

FIG. 7.

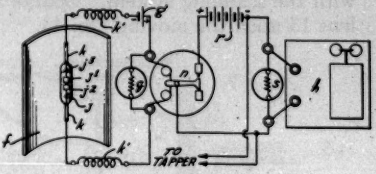

FIG. 8.

THE HOPEFUL PERIOD

The foregoing references to lenses and reflectors will be helpful in understanding some of the early disclosures of pictorial radio systems in the "hopeful period." We shall find that almost all of the proposals, practical and otherwise, required that the radio frequency energy be concentrated in a beam. Several patentees who followed Hertz, Fitzgerald, Lodge, Marconi, and the other pioneers in respect of time, also followed their teachings by using lenses or reflectors or both for concentrating radio frequency beams to obtain bi-dimensional information from reflecting objects.

A Pictorial Radio Proposal by Machts (1919)

Ludwig Machts, in a patent application filed in Germany on November 6, 1919, described "a device for detecting electrically conducting bodies hidden from the eye by non-conductors" (6). He was aware that electric waves emanating from a transmitter after reflection from conducting bodies had been picked up by a receiving device. Therefore, he proposed that a device be arranged to photograph from an airplane, earth covered metal deposits, coal, or water veins that reflected electric waves. Such waves, according to the patent, could be transmitted from and received by a device on the airplane and photographed on a film moving at a speed corresponding to the speed of the airplane.

Referring to Fig. 9, which corresponds to the German Patent drawing slightly modified, we find a transmitting antenna located within the cylindrical mirror 2. Electric waves from the antenna penetrate a non-conducting wall or region 4 and are reflected by the reflecting conductors 3. The reflected waves, after passing through the pitch lens 5, form an electric field image invisible to the eye. The image is scanned by a small spiral antenna 6 that rotates around the axis 7. As the antenna rotates, it is also moved radially in and out by suitable means schematically represented by the double arrow. Thereby the electric image area is scanned.

Currents, induced in the rotating antenna by way of the image field, are detected by a tube receiver 10 and are amplified by a pair of amplifier tubes 11 and 12. The amplified currents are applied to the spark gap 8. The spark gap is rotated and is moved radially in synchronism with the scanning system. Sparks at the gap are imaged by the glass lens 13 upon the moving film 14.

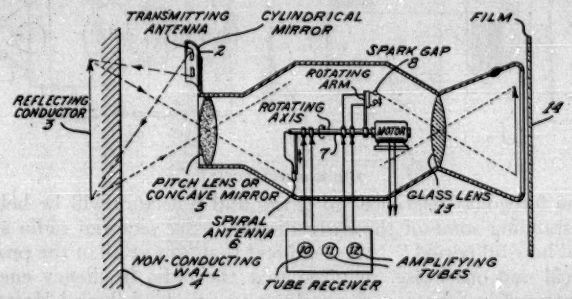

FIG. 9.

If shadings are desired, the patent suggests that several spark gaps be used to produce sparks of different brightness. Thus the reflected waves are imaged, converted into currents, amplified and detected, converted into light waves, and imaged on the film. The exposed film is developed and the observer derives from the differences in transparency of the film the arrangement and distribution of the wave reflecting electric conductors. A radio picture will be produced, provided the device operates as described in the patent.

Baird Pictorial System (1926)

John Logie Baird described a radio wave system for transmitting views or images to a distance in his United States Patent 1,699,270 (7). Figure 10, corresponding to Baird's patent drawing, shows a spark gap and dipole 2. Radio waves are radiated from the dipole and are focused by a lens 4 on object 6 to be viewed. Reflected waves are scanned by

an apertured and rotatable disc 8. A loop 10, responsive to the scanned waves, applies oscillating currents to a detector 12. The detected currents may be amplified by a device 14. The output of the amplifier 10 may be applied directly or by a modulated carrier wave to a receiver 16.

At the receiver the waves may be further amplified by an amplifier 18. The received amplified currents are applied to a glow-discharge

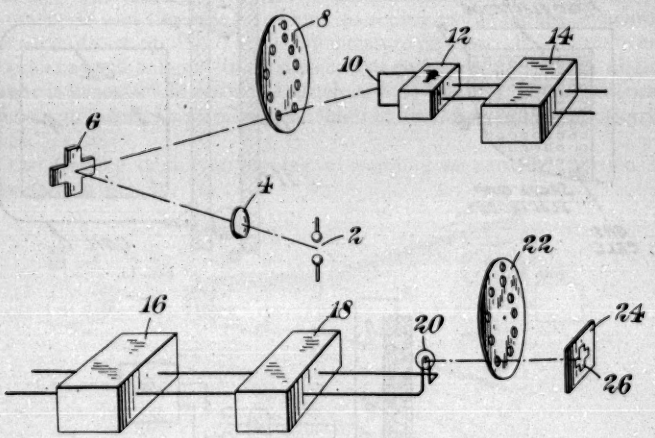

FIG. 10.

lamp 20. Light from the lamp is applied through the apertures in a scanning wheel 22 which is driven in synchronism with the disc 8. The light finally falls on a screen 24 to form an image 26 of the object 6.

Cawley Diavision (1921 and 1930)

Dr. Aloysius J. Cawley of Pittston, Pennsylvania, described in his United States Patent 2,083,292[4] several pictorial radio systems which he names "Diavision" (8). Since the patent specification includes many modifications, the most practical approach for the present purpose

[4] Also see Cawley, U. S. Patents 2,155,471 and 2,225,097 and abandoned patent application Serial No. 509,163.

In application Serial No. 509,163, filed September 17, 1921, and abandoned, Cawley showed the electrodes 6 connected in alternate rows by conductors 8 to the upper and lower bus bars 7, respectively. The Primary Examiner required a demonstration of the operability of the device. Cawley appealed to the Board of Appeals, who sustained the Primary Examiner. Cawley then appealed to the United States Court of Customs and Patent Appeals. The Court upon motion dismissed the appeal on March 25, 1931. By that time Cawley had filed a continuation-in-part application Serial No. 424,748 on which U. S. Patent 2,083,292 issued.

is to consider two species. One species requires no moving parts; the other uses a rotating scanner not unlike the device proposed by Ludwig Machts. Portions of the Cawley Patent Specification will be quoted to provide as nearly as practical the patentee's words. Reference is made to the patent specification for a complete description of the device and Dr. Cawley's theory of operation.

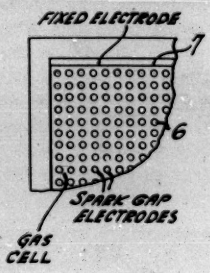

FIG. 11.

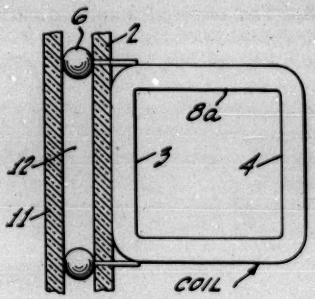

FIG. 13.

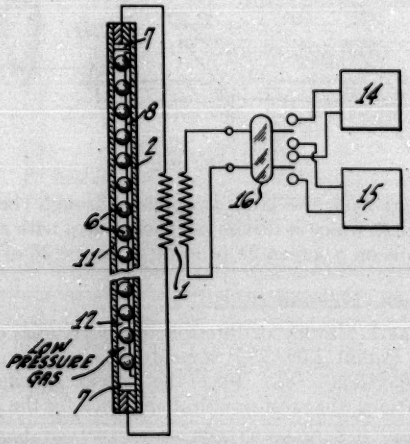

FIG. 12.

The Cawley device with stationary elements is illustrated in Figs. 11, 12, and 13, which correspond to Figs. 1, 2, and 4, respectively, of the patent.

Figure 11 "is an elevational view of the space in which the visible image is to be formed, showing the arrangement of the electrodes which are here illustrated as being unconnected, which detect the incoming

waves and render the gas luminous." Although the spark gap electrodes 6 and the fixed electrodes 7 are not shown connected in Fig. 11, the patentee may have had in mind the fixed electrode connections shown in Fig. 12, or the spark gap-resonator connections shown in Fig. 13. Perhaps he intended that no connections were required in the modification shown as Fig. 1 of the patent. In any event, the spheres 6 and the fixed electrodes 7 are positioned within a low pressure gas space 12. Power may be applied to the fixed electrodes 7 by means of the device 14 and the coils 1. Thus, according to the patent, synchronous currents are produced in the low pressure gas space 12. Moreover, the gas space 12 is kept "in such electrical condition that a very slight current is necessary in resonators such as 8(*a*) (see Fig. 13) to generate sufficient intensity to represent the 'high lights' of the image of electric waves."

The Cawley device employing a scanning or rotating portion is illustrated in Fig. 14.

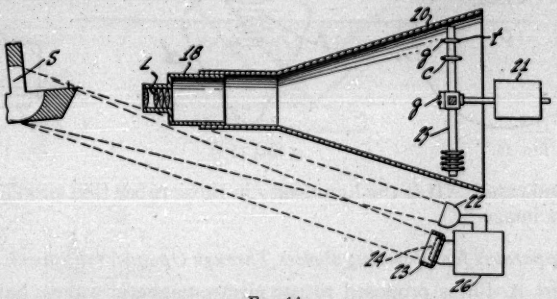

FIG. 14.

A high frequency generator 26 is connected to a plurality of Rhighi gaps 24. The gaps are positioned within "a cylindrical and parabolic" reflector 23 to establish a beam of polarized waves. In addition, the patentee includes "a spherical and parabolic" reflector 22 in which the generators (radiators) may be placed in various positions to emit and direct unpolarized waves. The reflectors 22, 23 may be made of metallic wires.

The waves are directed upon an object, such as ship *S*. "The waves are reflected from the conducting portions of the ship *S* into the lenses *L* of a camera-like structure 18 . . . , 20." The wave converting device illustrated in Figs. 11, 12, and 13 may be fitted into the end of the camera-like device. However, the patentee recommends the less expensive arrangement of a motor 21 and rotatable arms 25. Resonators *c*, each provided with a spark gap *g* and a neon tube *t*, are suitably positioned on the arms which are positioned at right angles to each other. In the described arrangement the resonator coils respond to unpolarized

waves. The response is indicated by the glowing of the neon tubes. Thus "the image is built up in a series of concentric circles."

In the event that the polarized wave generator 23 and 24 is used, the scanning device of Figs. 15, 15(*b*), and 15(*c*) (see Figs. 6, 6(*b*), and 6(*c*) of patent), is employed in the camera viewing device. "Here the visual field is represented by *f* somewhat similar to the general methods of television. It will be noticed that the coils *c* are shown with their spark gaps *g* in a horizontal . . . plane. That is, they traverse it in a definite direction, and their paths are straight lines. Arms or discs may be used. Figures 15, 15(*b*), and 15(*c*) show the disc *d* carrying a series of coils. Figure 15 represents the side of the disc that faces the lens *L* of Fig. 14. The spark gaps *g* are clearly illustrated . . . Fig. 15(*b*) shows the side of the disc facing the observer and the tubes *t* are

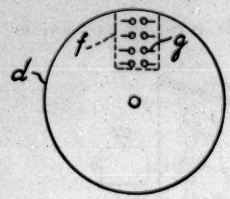

FIG. 15.

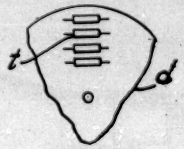

FIG. 15(*b*).

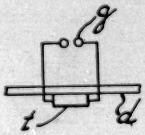

FIG. 15(*c*).

plainly indicated. It is the luminosity in those tubes that produces the luminous image."

Fliess Apparatus for Observing Bodies Through Opaque Substances (1930)

Robert A. Fliess proposed to use electromagnetic waves, having a wave length greater than infra red rays of light, by reflecting the waves from bodies hidden from unaided sight by opaque substances (9). The wave reflecting bodies might be at, under, or above the surface of the earth.

One form of the apparatus is illustrated by Figs. 16, 17, and 18 corresponding to Figs. 1, 2, and 3 respectively of Fliess United States Patent 2,075,808. As shown in Fig. 16, an ore deposit 2 capable of reflecting electromagnetic waves *R* is located below the surface of the earth 1.

A parabolic reflector 3 is arranged to direct a beam of electromagnetic waves downward in the direction of the deposit 2. A transmitter 4, designed for generating ultra-short electromagnetic waves, is connected with a radiating body 5 at the focus of the reflector, as shown in Fig. 17.

A belt 6 is mounted for rotation on a plurality of rollers, which may be four in number designated 7–7′, 8, 9. These rollers are arranged at the corners of a four-sided figure. One of these rollers, 8, has at its

base a gear 11 which meshes with a gear 12 on the shaft of a motor 13. The other end of the motor shaft is directly connected to a synchronizing commutator 14.

A multiple receiving antenna 15 is placed at a suitable distance from the transmitter. The antenna is connected with a detector 16 and an amplifier 17. The amplified current passes through the lead wires 18 and 19 into the oscillograph tube 20. The tube includes electrodes for

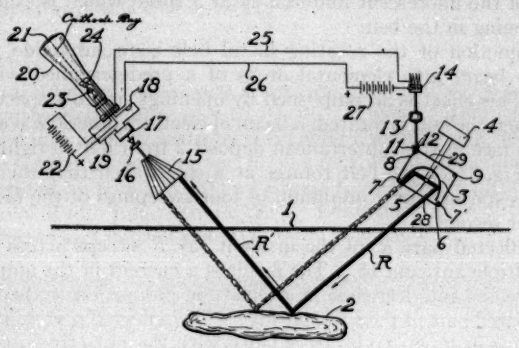

FIG. 16.

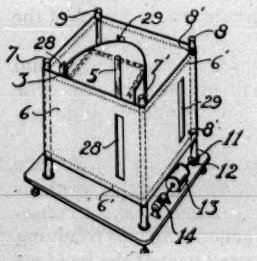

FIG. 17.

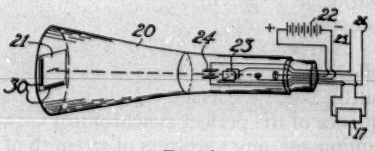

FIG. 18.

the production of a cathode ray. The ray impinges upon a fluorescent screen 21. A current for producing the cathode ray is supplied by the battery 22.

Two sets of deflector plates 23 and 24 (Figs. 16 and 18) are arranged in the oscillograph tube 20 along the axis of the cathode ray. One set of plates 24 is connected by lead wires 25 and 26 through a battery 27 to the synchronizing commutator 14. The other set of plates 23 receive currents from the lead wires 18 and 19 that are connected to the output of amplifier 17.

The motor 13 is directly connected to the driving roller 8 and the

commutator 14. The construction of the commutator is such that it closes the circuit through leads 25 and 26 whenever any one of the openings or slits 28 and 29 in the belt 6 starts to pass in front of the reflector opening. Therefore, the action of deflecting plates 24 is synchronized with the passage of a slit across the reflector opening. In other words, the progressively varying potential placed on the plates 24 tends to deflect the stream of electrons forming the cathode ray across the face of the fluorescent material 21 at a speed which is exactly that of the opening in the belt.

The function of the rotating metal belt is to subdivide the high frequency beam into elemental areas of a predetermined width and length. This effect is accomplished by openings suitably spaced in the belt. When the belt is rotated, a beam of electromagnetic waves sweeps across the face of the subterranean deposit 2 from left to right as indicated in Fig. 16. The belt rotates at a speed such that there are at least four exposures or a minimum of four sweepings of the face of the deposit per second.

The reflected part R' of the incident ray R sweeps across the face of the multiple antenna 15. The result is a current in the amplifier 17 which increases and decreases in intensity in proportion to the strength of the reflected part of the incident ray. The reflected part will increase and decrease in intensity in accordance with the reflective qualities of the subterranean deposit across which the beam is sweeping. The greater the reflective power of the deposit, the greater the strength of the reflected part of the incident ray.

The energy received by the antenna acts through the detector and amplifier upon a stream of electrons in the oscillograph tube 20. The variation in strength of the received energy deflects the cathode ray proportionately.

As shown in Fig. 18, a fluorescent line is traced across the surface of the screen 21. The resultant line indicates the reflective contour outline of the surface across which the beam is passing. As a consequence of the perfect synchronism between the generating and receiving equipment, any variation of strength of the reflected part of the incident ray may be located by means of a scale 30.

The patent describes a modification of the apparatus for searching for aircraft that are hidden from observers on the ground. The modified apparatus may be used to determine the contour and the velocity of the aircraft. In another modification, the system is proposed for use in aircraft to enable the operators to reach obscured landing fields and to record drift of the craft.

THE PRACTICAL PERIOD

Thus far we have traced the history of reflected radio waves through the pioneer and hopeful periods to the practical period in which a pic-

torial radio system was developed. It should be understood that there are no sharp dividing lines; overlap between speculation and practice is to be expected. Radio had developed rapidly during World War I. These developments became useful tools in the hands of scientists who began in 1925 to measure the height of the Heaviside Layer and who used radio to determine the altitude of aircraft (10). Continuous waves had almost completely taken the place of damped wave trains which had decayed in practice as in theory.

Wolff and Linder (1934)

Several watts of power were generated by a magnetron operated at wave lengths of the order of 9 to 10 cm. Waves of such length could be focused into a sharp beam by employing parabolic reflectors of practical size for the transmitter and for the receiver. This equipment was demonstrated by Irving Wolff and Ernest Linder before meetings of the Institute of Radio Engineers in 1934. Reflections were obtained from metal objects and ionized gas (11).

FIG. 19.

At the invitation of the Signal Corps, the apparatus (Fig. 19) was tested at Sandy Hook to determine its communication range and also to test its sensitivity for detecting reflections. Although higher power was desired to increase the initial range of about one-half mile, the 1934 demonstration with RCA experimental equipment showed that obstacles could not only be detected, but also that accurate azimuth and some elevation information could be obtained by observing the angular positions of the transmitted and reflected beams (12, 13). While the first tests were based upon audible signal output, graphic representations were within reach.

Tolson System for the Communication of Intelligence (1935)

Before proceeding with descriptions of several pictorial radio devices, reference will be made to a quasi-pictorial or graphical system invented by William A. Tolson. The Tolson system provides graphic indications by detecting on an aircraft receiver transmissions from radio beacons positioned at the boundaries of an airport. Figures 21 and 22, corresponding to Figs. 1 and 3 of Tolson United States Patent 2,130,913, illustrate, respectively, the system and one form of the aircraft receiving device (14).

Referring first to Fig. 20, the boundaries of field 10 are defined by a plurality of oscillators symbolically designated respectively *A*, *B*, *C*, and *D*. If, as is usually the case, the field is provided with boundary marking lights, the oscillators may each be conveniently mounted upon the light pedestals.

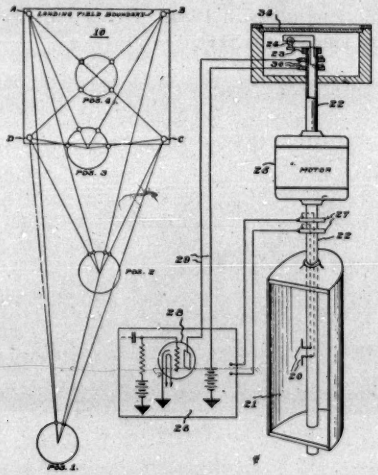

FIG. 20.

Oscillators *A*, *B*, *C*, and *D* are preferably driven by raw alternating current to provide 100 per cent modulation, and all operate at the same or approximately the same centimeter-wave frequency. The power output of each transmitter is preferably sufficient to operate a fairly insensitive receiver at a distance of, say, one mile.

The receiver as shown in Fig. 21 employs a cathode ray tube as the position indicating device. The tube 51 has magnetic deflecting coils 52 and 53 and a focusing anode 54 as well as an accelerating anode 55. The screen end 56 is coated internally with a suitable fluorescent mate-

rial. The tube is also provided with a cathode 57. A control electrode 59 is the other element in the tube; it may be biased to obtain either a "positive" or "negative" indication upon the screen 56. The biasing adjusting arrangement comprises simply a potentiometer 60 disposed across the terminals of a battery or other source of potential 61. Connections are made as shown with the grounded cathode 57. Anodes 54 and 55 are energized by direct current source 64 through potentiometer 65.

An antenna 71 and parabolic reflector 72, driven by a motor 73, are provided to pick-up successively energy radiated by the airport transmitters *A*, *B*, *C*, and *D*. The energy so collected passes through slip-rings 74 to a receiver 75. The output signals from the receiver are impressed through lead 76 upon the control electrode 59 of the tube 51.

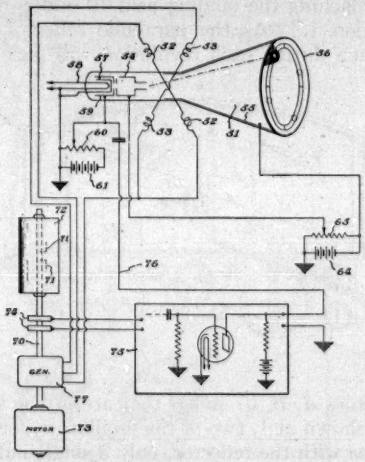

FIG. 21.

The deflecting coils 52 and 53 are energized by a small alternator 77 which may be conveniently mounted upon the common shaft 70. The alternator 77 frequently corresponds to the rotations of the shaft 70 driven by motor 73; that is, preferably 16 revolutions per second, or higher. In the absence of any bias or signal potential upon grid 59 the beam emanating from cathode 57 will rotate continuously about the inner circumference of the fluorescent screen 56 in synchronism with rotations of the antenna.

If the receiver 75 is so designed as to translate the energy periodically picked up by antenna 71 into separate negative voltage impulses of sufficient amplitude to cut-off the beam momentarily, the circle of

light around screen 56 will be broken. The number of breaks or "dark spots" and the position of these spots relative to each other correspond to the number and sequence of the stations *A*, *B*, *C*, and *D* picked up by antenna 71 in a single revolution. If, on the other hand, grid 59 is normally negatively biased to cut-off, the beam from cathode 57 will impinge upon screen 56 only when a positive voltage impulse, of sufficient amplitude, from receiver 75 is impressed upon the grid 59. Thus it is apparent that either a positive or negative (that is, "bright spot" or "dark spot") indication, whichever is desired, will appear upon fluorescent screen 56 to notify the observer of the position of his craft with respect to the remotely situated boundary marking transmitters *A*, *B*, *C*, and *D*.

Let us assume that an aircraft, carrying an indicator of the type described, is approaching the landing field 10 and is now at the point designated "position 1." As the parabolic reflector on the aircraft rotates, the antenna during each revolution will pick up signals from

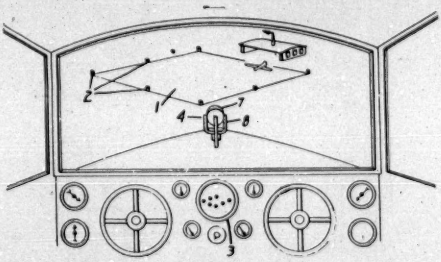

FIG. 22.

each of the oscillators *A*, *B*, *C*, and *D* that are in line with its plane of reception. If, as shown at 1, two of the oscillators, for instance *A* and *D*, are in direct line with the reflector, only a single indication for both boundary points will be registered upon the translucent screen. As the craft approaches position 2 the spots of light on the screen representing the boundaries of the landing field 10 begin to separate, thus giving the pilot a fairly accurate indication of his distance from the airport.

At position 2 the pilot may circle the airport if necessary, to assume a preferred landing direction, and while so engaged keep informed of the position and approximate distance of his craft with respect to the field. As the craft approaches the boundary of the airport, for example at position 3, the fact that two of the spots on the indicator are oppositely located indicates to the pilot that he is crossing the boundary of the field. Likewise, at position 4, the "spots" being symmetrically spaced on the screen, the pilot is informed that his craft is directly over the center of the field.

Becker Aircraft Landing System (1936)

A quasi-pictorial radio aid for aircraft landing was also proposed by Howard I. Becker in his United States Patent 2,151,549 (15).[5] Figures 1 and 2 of the Becker patent are shown here as Figs. 22 and 23, respectively.

The boundaries of an airport 1 are marked by short wave transmitters 2. The radiations from the transmitters are picked-up by a

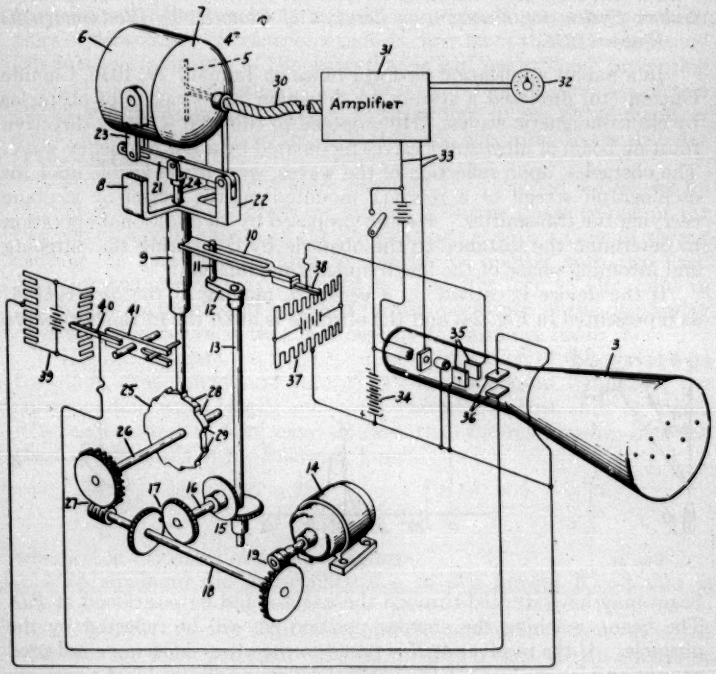

FIG. 23.

scanning antenna 4 mounted on the aircraft. The scanning is accomplished by swinging the antenna back and forth in the horizontal plane and up and down in the vertical plane. The electron beam of a cathode ray tube 3 is moved in synchronism with the antenna scanning by bias voltages derived from the potentiometers 37, 39 and applied to pairs of deflecting electrodes 35, 36, respectively.

Signals from the boundary marker transmitter 2 are received by the

[5] Also see Greig U. S. Patent 2,226,860.

dipole antenna 5 which is positioned within the reflector 7. The signals are applied to an amplifier 31. The amplified signals are impressed on a control electrode within the cathode ray tube to permit the electronic beam to reach the fluorescent screen of the tube. Thus, illuminated spots are established on the screen. The spot arrangement corresponds to the boundary markers and conveys "a picture of the landing area" to the pilot of the craft.

Gutton System for Detection or Location of Obstacles by Electromagnetic Waves (1936)

In a patent application filed in France on January 29, 1936, Camille Gutton (16) disclosed a system for detection and location of obstacles by electromagnetic waves. He proposed to transmit a highly directive rotating beam of ultra-short waves modulated by a low frequency wave. The obstacles, upon reflection of the waves, were made visible upon an oscillograph screen of a receiver mounted on the vessel or airplane carrying the transmitter. He also proposed by an additional operation to determine the distance to the obstacle by comparing the outgoing and incoming phase of the low frequency modulation.

If the device is carried by a vessel B, moving in the direction A, as represented in Fig. 24, and the obstacle is at O, the ultra-short wave

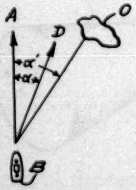

FIG. 24.

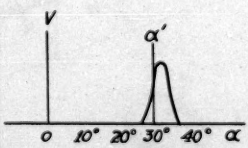

FIG. 25.

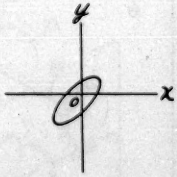

FIG. 26.

beam may have rotated through the angle α and be positioned at BD. The beam, reaching the angular position α', will be reflected by the obstacle. If the receiver at B is tuned to the ultra-short wave reflected energy, and is connected to the vertical deflecting electrodes of a cathode ray tube, the cathode ray will be deflected in the direction OV as shown in Fig. 25. Moreover, by deflecting the cathode ray in the direction $O\alpha$, in synchronism with the rotation of the outgoing beam, the angular position of the obstacle may be shown with reference to the course line BA. The angle subtended by the obstacle O will be indicated on the scale $O - \alpha$. The height of the vertical deflection is a clue to the distance to the obstacle.

According to Gutton's patent, the distance may be determined "more precisely by using as a basis the total travelling time, going and

coming, of the wave sent out from the vessel and reflected from the obstacle. Now, this interval of time is measurable in a particularly simple manner, for instance, by comparing the phase of the modulated current resulting from the return wave with that of the modulation of the transmitted wave. Also in this case this measurement of the phase angle or shift is very easily accomplishable as shall hereinafter be described by the use of a cathode-ray oscillograph. . . ."

Figure 26 "illustrates the form of the action or pattern that is observed upon the screen of the oscillograph by impressing upon both pairs of deflector electrodes potentials or currents of the same frequency (modulation frequency of the wave that is sent out) though presenting a phase displacement angle, . . . The orientation of the axes of this ellipse and its shape will vary, . . . according to the phase displacement angle of these two waves, that is to say, according to the distance of the obstacle. . . .

"In order to measure it more precisely . . . , action is brought by the aid of a calibrated corrector circuit, upon the phase of one of the two waves, say, the phase of the outgoing wave, with a view to always causing the axes of the ellipse to coincide or register with two fixed axes, for instance, the axes *ox* and *oy*" (see Fig. 27). "The value of this corrective phase shift will then furnish a criterion of the de-phasing caused by the distance, that is to say, by the distance itself.

"If d be this distance, v the speed of propagation of this wave, θ its travelling time (away and return), f the modulation frequency, the corresponding wave-length and ϕ the corrective phase shift or angle, it is, as a matter of fact, easy to show that the relationship between these quantities is of the following form:

$$d = \frac{v\theta}{2} = \frac{(2K-1)v}{8f} = v \cdot \phi = \left[\frac{\lambda}{4}\frac{2K-1}{2} = \frac{\phi}{\pi}\right],$$

where K is a positive or negative integer.

"To eliminate the uncertainty due to this integer K, all that is necessary is to choose a modulation wave-length sufficiently great so that it is always greater than four times the maximum distance at which obstacles are to be detected. Indeed, in such a case, K will always be equal to unity, and the distance is always determinable without ambiguity by this relation:

$$d = \frac{\lambda}{4}\left[\frac{1}{2} - \frac{\phi}{\pi}\right].$$

"To measure the distance d, it will, under these conditions, be only necessary to be able to vary the phase ϕ from $+\frac{\pi}{2}$ to $-\frac{\pi}{2}$.

"It will be understood that, . . . any other method may be employed which allows to utilize the form of the resultant ellipse. For

example, the ellipse could always be brought back so as to become or coincide with a straight line. The distance d in this latter case would be given by an entirely similar formula $d = \frac{\lambda}{4}\left(K - \frac{\phi}{\pi}\right)$, where K could have the two values 0 and 1."

The patent specification describes some of the details of the phasing circuit, the low-frequency oscillator-modulator, and the high frequency oscillator. Thus Gutton describes a system which gives graphic or pictorial information concerning obstacles along the course of a vessel or airplane. While the angular bearing is shown, and the angle subtended by the obstacle is indicated, the distance is determined by a separately adjusted phasing device or by a phase indicator.

To be continued.

REFERENCES FOR PART I

(1) HEINRICH HERTZ, "Electric Waves," *Annalen der Physik und Chemie* (*Wiedemann's Annalen*), Vol. 34, p. 155 (1888), translated by D. E. Jones, 1901.

(2) FRED T. TROUTON, "Repetition of Hertz's Experiments, and Determination of the Direction of the Vibration of Light," *Nature*, Vol. XXXIX, Feb. 21, 1889, pp. 391–393.

(3) OLIVER LODGE AND J. L. HOWARD, "On Electric Radiation and its Concentration by Lenses," (Physical Society, read May 11, 1889), *Phil. Mag.*, Vol. 28, p. 48 (1889).

(4) G. MARCONI, "Transmitting Electrical Impulses and Signals and Apparatus Therefor," U. S. Patent 586,193, issued July 13, 1897, filed December 7, 1896, reissue 11,913 granted June 4, 1901, "Principles of Electric Wave Telegraphy," 4th Edition, by FLEMING, pp. 449–451 (1919).

(5) G. MARCONI, "Radio Telegraphy," *Proc. I.R.E.*, Vol. 10, No. 4, August 1922, pp. 215–238, 226, 227.

(6) LUDWIG MACHTS, German Patent 330,090, "Vorrichtung zum Auffinden von elektresch leitenden, dem Auge durch Nichtleiter verborgenen Körpern," issued December 7, 1920, filed November 6, 1919.

(7) J. L. BAIRD, "Apparatus for Transmitting Views or Images to a Distance," U. S. Patent 1,699,270, issued January 15, 1929; filed in U. S. May 4, 1928; filed in Great Britain December 21, 1926; German Patent 522,487, Ausgegeben April 9, 1931.

(8) A. J. CAWLEY, "Diavision," U. S. Patent 2,083,292, issued June 8, 1937, filed January 30, 1930.

(9) R. A. FLIESS, "Method and Apparatus for Observing Bodies Through Opaque Substances," U. S. Patent 2,075,808, issued April 6, 1937, filed November 21, 1930.

(10) C. D. TUSKA, "Historical Notes on the Determination of Distance by Timed Radio Waves," JOURNAL FRANKLIN INST., Vol. 237, Nos. 1 and 2, January and February, 1944, pp. 1–20 and 83–102.

(11) I. WOLFF, E. G. LINDER, AND R. A. BRADEN, "Transmission and Reception of Centimeter Waves," *Proc. I.R.E.*, Vol. 23, No. 1, January 1935, p. 11.

(12) IRVING WOLFF, "The Story of Radar," *Radio Age*, pp. 8, 11, 12, 13 (1947).

(13) ROGER B. COLTON, "Radar in the United States Army," *Proc. I.R.E.*, Vol. 33, No. 11, pp. 740–753 (1945).

(14) W. A. TOLSON, "System for the Communication of Intelligence," U. S. Patent 2,130,913, issued September 20, 1933, filed April 30, 1935.

(15) H. I. BECKER, "Aircraft Landing Equipment," U. S. Patent 2,151,549, issued March 21, 1939, filed July 6, 1936.

(16) CAMILLE GUTTON, "Perfectionnement des moyens de décelement et de repérage des obstacles par ondes éléctromagnetiques," French Patent 812,975, filed January 29, 1936.

THE STEPPED SHAPE FILM APPLIED TO A JOURNAL BEARING

BY

FRANK R. ARCHIBALD [1]

The hydrodynamic theory of lubrication when applied to the ordinary journal bearing leads to mathematical problems of great difficulty. Though some progress has been made the problem is by no means as straightforward as in those cases where solutions have been obtained in the slipper bearing problem. It is the purpose here to show how the stepped film design suggested by Lord Rayleigh[2] might be applied to the journal bearing problem. Fairly simple explicit formulas will be obtained for certain arrangements of this film shape.

In the previously mentioned paper by Lord Rayleigh it is shown that if side leakage is neglected, the film shape for greatest load capacity in a simple slider bearing is the stepped shape of Fig. 1.

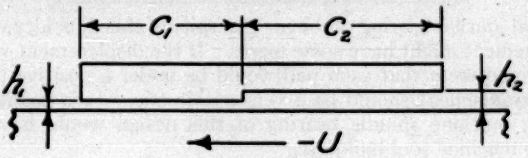

FIG. 1.

The present writer has shown[3] that if this shape is applied to a slider of finite width (that is, considering the effect of side leakage), the load supported can be given by an explicit formula in the form of a rapidly convergent series. The extension of this stepped film design to the case of the journal bearing is a natural one.

If the effect of curvature is neglected, the problem for the journal bearing, as far as pressure is concerned, is identical with the slider bearing problem when the shaft and bearing diameters are concentric as shown in Fig. 2. But such an arrangement as Fig. 2 could not run in a concentric position and carry any radial load. For if it is imagined to be running concentrically, it is in equilibrium under the resultant pressure loads of each stepped pad. An external radial load would cause a radial movement to a new position of equilibrium. This movement would have the effect of increasing the pressure on the side where

[1] Assistant Professor, School of Engineering, Princeton University, Princeton, N. J.

[2] Lord Rayleigh, "Notes on the Theory of Lubrication," *Phil. Mag.*, Vol. 35, pp. 1–12 (1918).

[3] F. R. Archibald, "A Simple Hydrodynamic Thrust Bearing," *Trans. ASME*, May 1950, paper No. 49-A-29.

the film thickness was reduced and lowering the pressure on the side where the film thickness was increased. The mathematical treatment of the problem in this displaced position would be at least as difficult

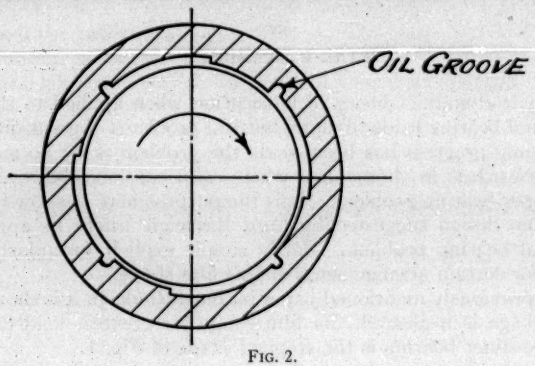

FIG. 2.

as the usual journal bearing problem. In spite of this difficulty, though, the arrangement might have some merit. If the displacement were not great it would seem that each pad would be under a positive pressure and the arrangement should be a very stable one. It is possible that a grinding machine spindle bearing of this design would have some interest to machine tool builders.

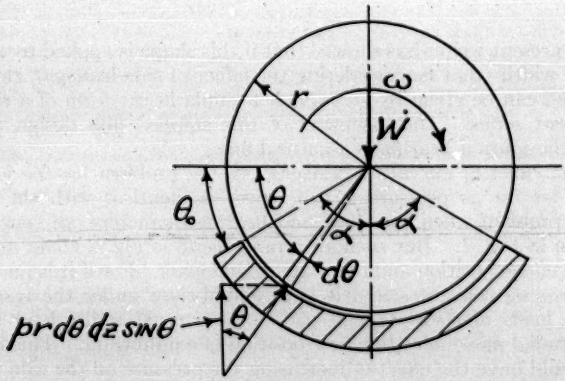

FIG. 3.

But it is with cases where concentricity under radial load can be, at least theoretically, obtained that we will be concerned here.

Consider the partial bearing of Fig. 3. By properly choosing the

conditions this bearing will run in a concentric position. This may be stated another way: if the dimensions shown in Fig. 3—together with a shaft speed, an oil viscosity and a bearing length—are chosen arbitrarily, the shaft will run concentrically with a certain load. This load will now be obtained.

A stepped slider bearing of finite width is shown in Fig. 4.

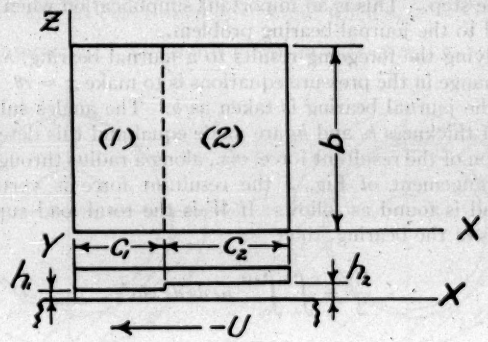

FIG. 4.

The writer has shown[3] that the pressure in region (1) is given by:

$$p = \sum_{n=1,3,5\cdots}^{\infty} \frac{P_n}{\sinh \frac{n\pi c_1}{b}} \sin \frac{n\pi z}{b} \sinh \frac{n\pi x}{b}. \tag{1}$$

For region (2) it is convenient to move the Z-axis forward to the leading edge of the slider. The pressure at any point in region (2) is then given by:

$$p = -\sum_{n=1,3,5\cdots}^{\infty} \frac{P_n}{\sinh \frac{n\pi c_2}{b}} \sin \frac{n\pi z}{b} \sinh \frac{n\pi x}{b}, \tag{2}$$

where in both cases the Fourier coefficient P_n is given by:

$$P_n = \frac{24\mu U b(h_2 - h_1)}{n^2\pi^2 \left(h_1^3 \coth \frac{n\pi c_1}{b} + h_2^3 \coth \frac{n\pi c_2}{b} \right)}. \tag{3}$$

The load, W, supported by slider is found by integrating the pressure over the areas (1) and (2). This gives:

$$W = \frac{48\mu U b^3(h_2 - h_1)}{\pi^4} \sum_{n=1,3,5\cdots}^{\infty} \frac{1}{n^4} \frac{\tanh \frac{n\pi c_1}{2b} + \tanh \frac{n\pi c_2}{2b}}{h_1^3 \coth \frac{n\pi c_1}{b} + h_2^3 \coth \frac{n\pi c_2}{b}}. \tag{4}$$

It is found by trial that the load capacity for a square slipper is a maximum when $h_2 = 1.7h_1$ and $c_2 = 1.2c_1$, but the change in load capacity is very small for large variations of these proportions. For all practical purposes the maximum load for a square slipper is obtained with $c_1 = c_2$. If the bearing is made this way, the pressure diagrams for regions (1) and (2) are the same and the resultant load passes through the step. This is an important simplification when the results are applied to the journal bearing problem.

In applying the foregoing results to a journal bearing, as in Fig. 3, the only change in the pressure equations is to make $x = r\theta$. The axial length of the journal bearing is taken as b. The angles subtended by the films of thickness h_1 and h_2 are made equal and this determines the line of action of the resultant force, *viz.*, along a radius through the step. In the arrangement of Fig. 3 the resultant force is vertical. This vertical load is found as follows: If W is the total load supported by both halves of the bearing, then

$$\frac{W}{2} = \int_0^b \int_{\theta_0}^{\pi/2} pr\, d\theta\, dz \sin\theta. \tag{5}$$

The pressure, p, can be written down from Eq. 1 making the appropriate changes in the X-coordinate.

$$p = \sum_{n=1,3,5\cdots}^{\infty} \frac{P_n}{\sinh \dfrac{n\pi r\alpha}{b}} \sin \frac{n\pi z}{b} \sinh \frac{n\pi r}{b} (\theta - \theta_0), \tag{6}$$

and

$$P_n = \frac{24\mu r\omega b(h_2 - h_1)}{n^2\pi^2(h_1^3 + h_2^3) \coth \dfrac{n\pi r\alpha}{b}}.$$

Then

$$W = 2\int_0^b \int_{\theta_0}^{\pi/2} \sum_{n=1,3,5\cdots}^{\infty} \frac{P_n}{\sinh \dfrac{n\pi r\alpha}{b}} \sin \frac{n\pi z}{b} \sinh \frac{n\pi r}{b} (\theta - \theta_0) \sin\theta r\, d\theta\, dZ$$

$$= 2\int_{\theta_0}^{\pi/2} \sum_{n=1,3,5\cdots}^{\infty} \frac{2rbP_n}{n\pi \sinh \dfrac{n\pi r\alpha}{b}} \sinh \frac{n\pi r}{b} (\theta - \theta_0) \sin\theta\, d\theta.$$

Therefore

$$W = 4r^2 \sum_{n=1,3,5\cdots}^{\infty} \frac{P_n}{\left(\dfrac{n^2\pi^2r^2}{b^2} + 1\right) \sinh \dfrac{n\pi r\alpha}{b}} \left(\cosh \frac{n\pi r\alpha}{b} - \sin\theta_0\right). \tag{8}$$

Placing in the value of P_n from Eq. 7,

$$W = \frac{96\mu\omega r^3 b(h_2 - h_1)}{\pi^2(h_1^3 + h_2^3)} \sum_{n=1,3,5\cdots}^{\infty} \frac{\cosh \frac{n\pi r\alpha}{b} - \sin\theta_0}{n^2\left(\frac{n^2\pi^2r^2}{b^2} + 1\right)\cosh \frac{n\pi r\alpha}{b}}. \quad (9)$$

The two special cases when $\theta_0 = 0$ and when $\theta_0 = -\frac{\pi}{2}$ are perhaps the most interesting. These are shown in Figs. 5 and 6.

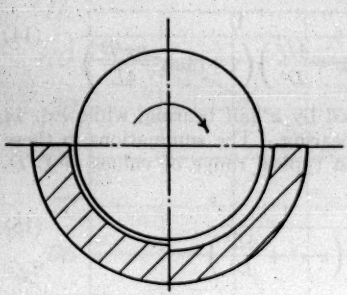

FIG. 5.

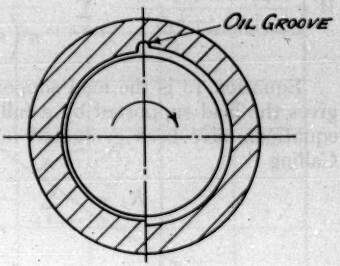

FIG. 6.

First consider the case of $\theta_0 = 0$. This is a half bearing and here $\alpha = \frac{\pi}{2}$. The load formula becomes

$$W = \frac{96\mu\omega r^3 b(h_2 - h_1)}{\pi^2(h_1^3 + h_2^3)} \sum_{n=1,3,5\cdots}^{\infty} \frac{1}{n^2\left(\frac{n^2\pi^2r^2}{b^2} + 1\right)}. \quad (10)$$

For the case of the full bearing $\theta_0 = -\frac{\pi}{2}$ and $\alpha = \pi$, the load formula becomes

$$W = \frac{96\mu\omega r^3 b(h_2 - h_1)}{\pi^2(h_1^3 + h_2^3)} \sum_{n=1,3,5\cdots}^{\infty} \frac{\cosh \frac{n\pi^2 r}{b} + 1}{n^2\left(\frac{n^2\pi^2r^2}{b^2} + 1\right)\cosh \frac{n\pi^2 r}{b}}. \quad (11)$$

This last equation is not in a form very suitable for computation, but by making use of the well known relations for hyperbolic functions it can be altered to

$$W = \frac{192\mu\omega r^3 b(h_2 - h_1)}{\pi^2(h_1^3 + h_2^3)} \sum_{n=1,3,5\cdots}^{\infty} \frac{1}{n^2\left(\frac{n^2\pi^2r^2}{b^2} + 1\right)\left(1 + \tanh^2 \frac{n\pi^2 r}{2b}\right)}. \quad (12)$$

In Eqs. 10 and 12 the ratio r/b occurs. It is more convenient to have the diameter of the bearing rather than its radius appearing and the ratio L/D is more usual in journal bearing theory. Also the rotational speed, N, in revolutions per minute is more suitable. If these notations are substituted in Eqs. 10 and 12 they become

$$W = \frac{8\mu NDL^3(h_2 - h_1)}{5\pi(h_1^3 + h_2^3)} \sum_{n=1,3,5\cdots}^{\infty} \frac{1}{n^2\left(n^2\pi^2 + \frac{4L^2}{D^2}\right)} \tag{13}$$

$$W = \frac{16\mu NDL^3(h_2 - h_1)}{5\pi(h_1^3 + h_2^3)} \sum_{n=1,3,5\cdots}^{\infty} \frac{1}{n^2\left(n^2\pi^2 + \frac{4L^2}{D^2}\right)\left(1 + \tanh^2\frac{n\pi^2 D}{4L}\right)}. \tag{14}$$

Equation 13 is the load supported by a half bearing, while Eq. 14 gives the load supported by a full bearing. The summations in these equations have been evaluated for a typical range of values of L/D. Calling

$$K_1 = \sum_{n=1,3,5\cdots}^{\infty} \frac{1}{n^2\left(n^2\pi^2 + \frac{4L^2}{D^2}\right)}, \tag{15}$$

$$K_2 = \sum_{n=1,3,5\cdots}^{\infty} \frac{1}{n^2\left(n^2\pi^2 + \frac{4L^2}{D^2}\right)\left(1 + \tanh^2\frac{n\pi^2 D}{4L}\right)}. \tag{16}$$

A pair of curves shows the value of K_1 and K_2 in Fig. 7.

The factor $\frac{h_2 - h_1}{h_1^3 + h_2^3}$ which appears in the load equations is easily shown to have a maximum when $h_2 \cong 1.68h_1$. Thus for the conditions of maximum load, Eqs. 13 and 14 can be written

$$W = .06033\,\frac{\mu NDL^3}{h_1^2}K_1 \quad \text{for half bearing} \tag{17}$$

$$W = .1207\,\frac{\mu NDL^3}{h_1^2}K_2 \quad \text{for full bearing.} \tag{18}$$

It is desirable to make comparisons of this bearing to one of conventional design but unfortunately there does not seem to be a very good way in which this can be done. To begin with, the stepped film bearing does not have a single figure which can be called the clearance. If minimum film thicknesses are taken equal for the comparison it is not clear what clearance or what eccentricity should be used in the conventional bearing. The author has not done much to resolve this dilemma. A few trials with roughly comparable figures have shown that the stepped bearing has about the same theoretical capacity as a bearing

of conventional design. This is not surprising in view of the results of the stepped slipper bearing. Here the load capacity is only slightly more than for the inclined plate slipper bearing if the slipper is square.

Another matter of practical concern is how such a bearing is to be made. In the case of the stepped slipper bearing all that is required is two flat surfaces giving the slight step. This can be done by surface grinding which is one of the easiest and most accurate machine shop operations. But a cylinder having a stepped diameter is not an espe-

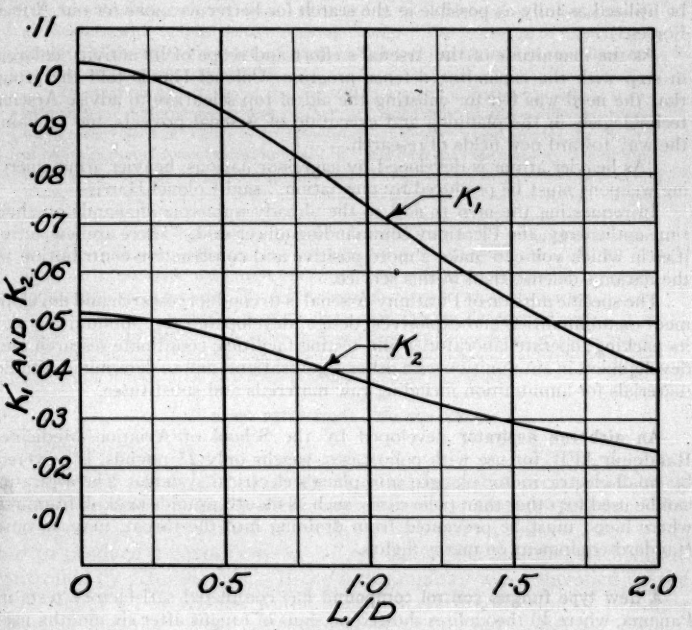

FIG. 7.

cially easy surface to produce. It is possible that it might be scraped to fit a stepped bar which itself might be made by grinding and partial plating. A controlled sand blast might also be used. However, the use of etching seems to offer the best solution to the problem. Undoubtedly this practical matter of how best to get the desired shape is a serious obstacle to the use of the stepped film journal bearing. But it does not seem an insurmountable difficulty and the author has not considered it a sufficient reason for not giving the foregoing results to those who may find them of interest.

Ordnance Advisory Council.—Eighteen of the nation's outstanding men in the fields of physics, electronics, metallurgy, chemistry and engineering have agreed to serve as members of a Scientific Advisory Council to Picatinny Arsenal, Dover, N. J., the Army Ordnance Corps' principal installation for research and development in the field of propellants and explosives, artillery ammunition above 20 mm, bombs, pyrotechnics and associated items.

In greeting the Council members at their initial meeting Colonel J. P. Harris, Picatinny's Commanding Officer, told them, "The present world situation requires more than ever before that the talents of the nation's scientists be utilized as fully as possible in the search for better weapons for our Armed Forces."

As the magnitude of the Arsenal's effort and scope of its activity enlarge, in step with the expanding defense program, Colonel Harris told the group that the need was felt for enlisting the aid of top scientists to advise Arsenal technologists in the planning and execution of Arsenal projects and to point the way toward new fields of research.

"As heavier armor is developed by aggressor nations, heavier armor-piercing weapons must be produced by our nation," said Colonel Harris.

In requesting the men to add to the already numerous demands on their time and energy, the Picatinny commanding officer said, "There are few activities in which you can make a more positive and constructive contribution to the nation's defense than in this service."

The specific mission of Picatinny Arsenal is to conduct research and development on ammunition and explosives; design, develop and test ammunition and its packing; operate laboratories and testing facilities; coordinate research and development in ammunition with other organizations, and to determine suitable materials for ammunition including raw materials and substitutes.

An airborne aspirator developed by the School of Aviation Medicine, Randolph AFB, for use with polio cases, weighs only 25 pounds, is powered by small electric motor plugged into plane's electrical system. The aspirator can be used for other than polio cases, such as mouth wounds or skull fractures where blood must be prevented from draining into the throat, may become standard equipment on mercy flights.

A new type fungus control compound has completed satisfactory tests in Panama, where 40 theodolites showed no signs of fungus after six months use. The compound, consisting of beeswax, vaseline and mercury poison, was developed by the Inter-American Geodetic Survey, of the Army's Caribbean Headquarters, and is expected to save thousands of dollars by reducing repair work on intricate instruments used in damp, humid climates.

A six sided "hex tent" now being shipped to Korea represents last word in refinement of the World War II model pup tent. Each tent shelters five men in comfort at temperatures down to 95 degrees below freezing. The tents resemble tepees, with ground cloth to keep out cold wind, seal in warm air provided by small gasoline Yukon stove. Light aluminum tent pegs and telescoping center pole help keep overall weight down to 56 pounds.

A THEORY OF MULTIDIMENSIONAL SERVO SYSTEMS*

BY

MICHAEL GOLOMB[1] AND EUGENE USDIN[1]

ABSTRACT

An n-dimensional servo system is defined as a combination of n simple servomechanisms with coupled error-measuring devices. The system is to reproduce n inputs, and the controlling feedbacks consist of linear combinations of the differences between the n inputs and outputs.

The analysis undertaken in this paper is to supply the designer of such systems with general performance criteria. They are essentially of three different types. First, error coefficient matrices are established which give a measure for the quality of reproduction of some typical inputs. In the second part the stability of n-dimensional servo systems is investigated with special emphasis on the possibilities of combining stable and unstable servos into stable systems by judicious coupling. There are several results to the effect that this is possible, and that the time delay of the response can be diminished in most cases where there is a sufficient number of design parameters (including the coupling coefficients) at the designer's disposal. In the third part, rms.-error and integrated-square error criteria are established for multidimensional servo systems. By developing expressions in which these errors appear formally identical with the corresponding errors for one-dimensional servos, it is shown how the design techniques developed for one-dimensional servos, such as attenuation-phase diagrams, can be applied to multidimensional systems whenever theoretical minimization of the errors is impossible or impractical.

I. BASIC DEFINITIONS AND EQUATIONS

1. *Introduction*

An n-dimensional servo system may be defined as a combination of n simple servomechanisms with coupled error-measuring devices. The n components are assumed to receive n arbitrary inputs $x_1, x_2, \cdots, x_n$ and to produce n outputs $y_1, y_2, \cdots, y_n$ through n amplifiers which are controlled by error signals. If the k-th amplifier was controlled by the error signal $(x_k - y_k)$ one would simply have a collection of n simple servomechanisms. This collection becomes an n-dimensional system, when each or some of the amplifiers are controlled by combinations of error signals involving several of the differences $(x_k - y_k)$. In the following it will be tacitly assumed that these composite error signals are *linear* combinations of the differences $(x_k - y_k)$, an assumption necessitated by the present stage of development of the theory.

The combination of several servomechanisms into one multidimensional system is desirable when it leads to simpler and more accurate

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instrumentation. For example, a steering or a guiding control mechanism may be required to reproduce the three components of a vector quantity (velocity, acceleration, etc.) along the given axes A_1, A_2, A_3, but the installation requirements may be such that the error-measuring devices produce signals representing the component errors along three axes, B_1, B_2, B_3, which make fixed or slowly changing angles with the axes A_1, A_2, A_3. In this case the component errors along axes B_1, B_2, B_3, which are used to control the amplifiers, are linear combinations of the component errors along the axes A_1, A_2, A_3, which measure the differences of the outputs and inputs. Thus one is led to a three-dimensional servo system, which may be greatly superior to a battery of three one-dimensional servos.

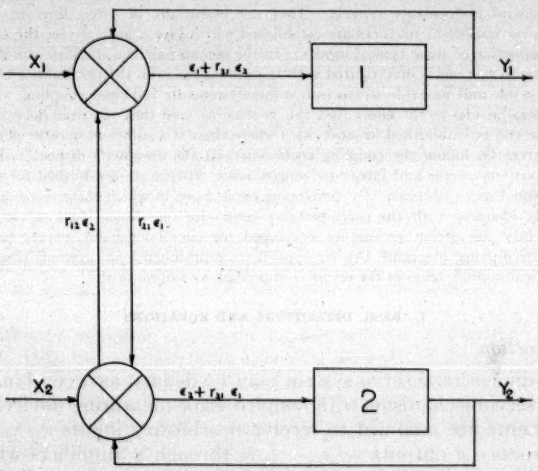

FIG. 1.

2. The System Equations

Figures 1 and 2 schematically show a two-dimensional and a three-dimensional servo system, respectively. The inputs $x_1(t), x_2(t), \cdots$, and outputs $y_1(t), y_2(t), \cdots$ are fed into error-measuring devices or comparators shown as crosses, where signals are produced that are linear combinations of the individual errors $\epsilon_1(t) = x_1(t) - y_1(t)$, $\epsilon_2(t) = x_2(t) - y_2(t), \cdots$. These signals control the outputs through systems of amplifiers, motors, and other devices, shown as boxes.

Inputs and outputs of an n-dimensional servo are related by a system of n linear differential equations which are easily established by the methods of mechanical and electrical circuit analysis. We assume that the signals from the comparators to the amplifiers are the following

linear combinations of the individual errors

$$\begin{aligned} &\epsilon_1(t) + r_{12}\epsilon_2(t) + \cdots + r_{1n}\epsilon_n(t) \\ &r_{21}\epsilon_1(t) + \epsilon_2(t) + \cdots + r_{2n}\epsilon_n(t) \\ &\cdots \\ &r_{n1}\epsilon_1(t) + r_{n2}\epsilon_2(t) + \cdots + \epsilon_n(t), \end{aligned} \tag{1}$$

where the r_{jk} are constants, called *coupling coefficients*. Then letting D represent the differential operator d/dt the equations are of the form

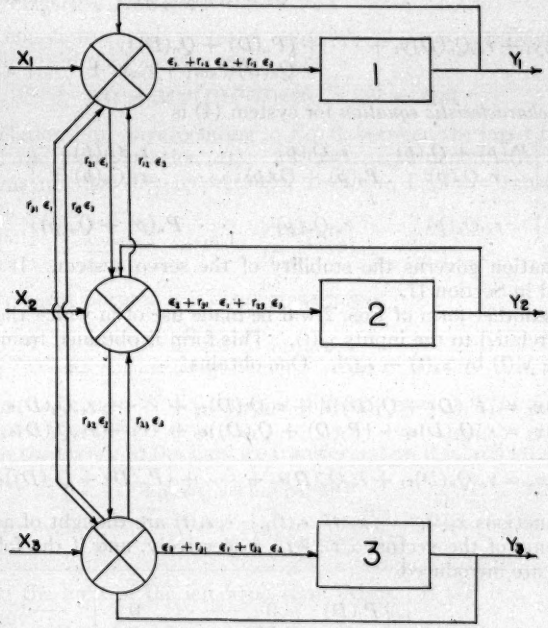

FIG. 2.

$$\begin{aligned} P_1(D)y_1 &= Q_1(D)(\epsilon_1 + r_{12}\epsilon_2 + \cdots + r_{1n}\epsilon_n) \\ P_2(D)y_2 &= Q_2(D)(r_{21}\epsilon_1 + \epsilon_2 + \cdots + r_{2n}\epsilon_n) \\ &\cdots \\ P_n(D)y_n &= Q_n(D)(r_{n1}\epsilon_1 + r_{n2}\epsilon_2 + \cdots + \epsilon_n), \end{aligned} \tag{2}$$

where $P_k(p)$, $Q_k(p)$ are rational polynomials of a complex variable p with real coefficients. The rational functions

$$Y_1(p) = \frac{Q_1(p)}{P_1(p)}, \; Y_2(p) = \frac{Q_2(p)}{P_2(p)}, \; \cdots, \; Y_n(p) = \frac{Q_n(p)}{P_n(p)} \tag{3}$$

are known as the *feedback transfer functions* of the individual servos. They relate the signals from the comparators to the outputs.

When the error functions $\epsilon_k(t)$ are replaced by $y_k(t) - x_k(t)$ another useful form of Eqs. 2 is obtained:

$$\begin{aligned}
&(P_1(D) + Q_1(D))y_1 + r_{12}Q_1(D)y_2 + \cdots + r_{1n}Q_1(D)y_n \\
&\qquad = Q_1(D)(x_1 + r_{12}x_2 + \cdots + r_{1n}x_n) \\
&r_{21}Q_2(D)y_1 + (P_2(D) + Q_2(D))y_2 + \cdots + r_{2n}Q_2(D)y_n \\
&\qquad = Q_2(D)(r_{21}x_1 + x_2 + \cdots + r_{2n}x_n) \qquad (4) \\
&\cdots \\
&r_{n1}Q_n(D)y_1 + r_{n2}Q_n(D)y_2 + \cdots + (P_n(D) + Q_n(D))y_n \\
&\qquad = Q_n(D)(r_{n1}x_1 + r_{n2}x_2 + \cdots + x_n).
\end{aligned}$$

The *characteristic equation* for system (4) is

$$\Delta(p) \equiv \begin{vmatrix} P_1(p) + Q_1(p) & r_{12}Q_1(p) & \cdots & r_{1n}Q_1(p) \\ r_{21}Q_2(p) & P_2(p) + Q_2(p) & \cdots & r_{2n}Q_2(p) \\ \cdots & & & \\ r_{n1}Q_n(p) & r_{n2}Q_n(p) & \cdots & P_n(p) + Q_n(p) \end{vmatrix} = 0. \qquad (5)$$

This equation governs the stability of the servo system. It will be discussed in Section II.

Still another form of Eqs. 2 will be made use of in which the errors $\epsilon_k(t)$ are related to the inputs $x_k(t)$. This form is obtained from (2) by replacing $y_k(t)$ by $x_k(t) - \epsilon_k(t)$. One obtains

$$\begin{aligned}
P_1(D)x_1 &= (P_1(D) + Q_1(D))\epsilon_1 + r_{12}Q_1(D)\epsilon_2 + \cdots + r_{1n}Q_1(D)\epsilon_n \\
P_2(D)x_2 &= r_{21}Q_2(D)\epsilon_1 + (P_2(D) + Q_2(D))\epsilon_2 + \cdots + r_{2n}Q_2(D)\epsilon_n \\
&\cdots \\
P_n(D)x_n &= r_{n1}Q_n(D)\epsilon_1 + r_{n2}Q_n(D)\epsilon_2 + \cdots + (P_n(D) + Q_n(D))\epsilon_n.
\end{aligned} \qquad (6)$$

If the functions $x_1(t), \cdots, x_n(t)$; $\epsilon_1(t), \cdots, \epsilon_n(t)$ are thought of as the n components of the vectors $\bar{x}(t)$, $\bar{\epsilon}(t)$, respectively, and if the following matrices are introduced

$$\mathcal{P}(D) = \begin{vmatrix} P_1(D) & 0 & \cdots & 0 \\ 0 & P_2(D) & \cdots & 0 \\ \cdots & & & \\ 0 & 0 & \cdots & P_n(D) \end{vmatrix}$$

$$\mathfrak{M}(D) = \begin{vmatrix} P_1(D) + Q_1(D) & r_{12}Q_1(D) & \cdots & r_{1n}Q_1(D) \\ r_{21}Q_2(D) & P_2(D) + Q_2(D) & \cdots & r_{2n}Q_2(D) \\ \cdots & & & \\ r_{n1}Q_n(D) & r_{n2}Q_n(D) & \cdots & P_n(D) + Q_n(D) \end{vmatrix}$$

then Eq. 6 may be rewritten as

$$\mathcal{P}(D)\bar{x} = \mathfrak{M}(D)\bar{\epsilon} \qquad (7)$$

and the formal solution of (7) for $\bar{\epsilon}$ is

$$\bar{\epsilon} = \mathfrak{M}^{-1}(D)\mathfrak{P}(D)\bar{x}. \tag{8}$$

The matrix

$$\mathfrak{R}(p) = \mathfrak{M}^{-1}(p)\mathfrak{P}(p) \tag{9}$$

will be referred to as *loop impedance matrix*. It will be made use of in Section III.

3. Error Coefficient Matrices

For one-dimensional servos a sequence of error coefficients has been defined[2] in the following way. Let

$$\epsilon(t) = M^{-1}(D)P(D)x(t), \qquad D = d/dt \tag{10}$$

be the relationship, corresponding to Eq. 8, between the input function $x(t)$ and the error function $\epsilon(t)$. Let $x(p)$, $\epsilon(p)$ denote the Laplace transforms of $x(t)$, $\epsilon(t)$, respectively, then the Laplace transform of Eq. 10 is

$$\epsilon(p) = \frac{P(p)}{M(p)}\,x(p). \tag{11}$$

For the present purpose it is convenient to put

$$\frac{P(p)}{M(p)} = K^{(s)}p^sR(p), \tag{12}$$

where s is an integer, $K^{(s)}$ a constant different from 0, $R(p)$ a rational function for which $R(0) = 1$.

From the theory of the Laplace transformation it is well known that

$$\begin{aligned}\lim_{t\to\infty}\epsilon(t) &= \lim_{p\to 0}p\epsilon(p)\\ &= K^{(s)}\lim_{p\to 0}p^{s+1}x(p)\end{aligned} \tag{13}$$

provided the limit of the left-hand term exists. If $x(t)$ is a "position step input"

$$x(t) = \begin{cases}0 & \text{for } t < 0\\ x^{(0)} & \text{for } t > 0\end{cases}$$

then $x(p) = x^{(0)}p^{-1}$, and Eq. 13 gives a finite static (or position) error

$$\lim_{t\to\infty}\epsilon(t) = K^{(0)}x^{(0)}$$

if $s = 0$, and zero static error if $s > 0$. $K^{(0)}$ is called the *static error coefficient.*

[2] H. M. James, N. B. Nichols and R. S. Phillips, editors, "Theory of Servomechanisms," New York, McGraw-Hill Book Co., Inc., 1947.

For a zero static error servo we may consider a "velocity step input"

$$x(t) = \begin{cases} 0 & \text{for } t < 0 \\ x^{(1)}t & \text{for } t > 0. \end{cases}$$

Then $x(p) = x^{(1)}p^{-2}$ and Eq. 13 gives a finite velocity error

$$\lim_{t\to\infty} \epsilon(t) = K^{(1)}x^{(1)}$$

if $s = 1$, and zero velocity error if $s > 1$. $K^{(1)}$ is the *velocity error coefficient.*

For a zero velocity error servo we may consider an "acceleration step input"

$$x(t) = \begin{cases} 0 & \text{for } t < 0 \\ x^{(2)}t^2/2 & \text{for } t > 0. \end{cases}$$

Then $x(p) = x^{(2)}p^{-3}$ and Eq. 13 gives a finite acceleration error

$$\lim_{t\to\infty} \epsilon(t) = K^{(2)}x^{(2)}$$

if $s = 2$, and zero acceleration error if $s > 2$. $K^{(2)}$ is the *acceleration error coefficient.*

Continuing in an obvious fashion one is thus led to a sequence of error coefficients $K^{(0)}, K^{(1)}, K^{(2)}, \cdots$, which are figures of merit for the servo measuring the steady-state errors of the responses to typical inputs.

It will now be shown how these error coefficients find their analogues in error coefficient matrices in the case of multidimensional servo systems. If $\bar{x}(p)$ is the n-vector whose components are the Laplace transforms $x_i(p)$ of the input functions $x_i(t)$ and $\bar{\epsilon}(p)$ is the n-vector whose components are the Laplace transforms $\epsilon_i(p)$ of the error functions then

$$\bar{\epsilon}(p) = \mathfrak{M}^{-1}(p)\mathfrak{O}(p)\bar{x}(p) \tag{14}$$

is the Laplace transform of Eq. 8. It is convenient to put

$$\frac{1}{1 + Y_i(p)} = \frac{P_i(p)}{P_i(p) + Q_i(p)} = K_i^{(s_i)}p^{s_i}R_i(p) \quad (i = 1, 2, \cdots, n), \tag{15}$$

where the s_i are integers, $K^{(s_i)}$ are constants different from 0, and the $R_i(p)$ are rational functions for which $R_i(0) = 1$.

The matrix $\mathfrak{M}(p)$ can be written as the following product

$$\mathfrak{M} = \begin{bmatrix} P_1+Q_1 & 0 & \cdots & 0 \\ 0 & P_2+Q_2 & \cdots & 0 \\ \cdots & & & \\ 0 & 0 & \cdots & P_n+Q_n \end{bmatrix} \times \begin{bmatrix} 1 & r_{12}\frac{Q_1}{P_1+Q_1} & \cdots & r_{1n}\frac{Q_1}{P_1+Q_1} \\ r_{21}\frac{Q_2}{P_2+Q_2} & 1 & \cdots & r_{2n}\frac{Q_2}{P_2+Q_2} \\ \cdots & & & \\ r_{n1}\frac{Q_n}{P_n+Q_n} & r_{n2}\frac{Q_n}{P_n+Q_n} & \cdots & 1 \end{bmatrix}.$$

Therefore

$$\mathfrak{M}^{-1} = \begin{bmatrix} 1 & r_{12}\frac{Q_1}{P_1+Q_1} & \cdots & r_{1n}\frac{Q_1}{P_1+Q_1} \\ \cdots & & & \\ \cdots & & & \\ r_{n1}\frac{Q_n}{P_n+Q_n} & \cdots & & 1 \end{bmatrix}^{-1} \times \begin{bmatrix} \frac{1}{P_1+Q_1} & 0 & \cdots & 0 \\ 0 & \frac{1}{P_2+Q_2} & \cdots & 0 \\ \cdots & & & \\ 0 & 0 & \cdots & \frac{1}{P_n+Q_n} \end{bmatrix}$$

and

$$\mathfrak{M}^{-1}\mathfrak{P} = \begin{bmatrix} 1 & r_{12}\frac{Q_1}{P_1+Q_1} & \cdots & r_{1n}\frac{Q_1}{P_1+Q_1} \\ \cdots & & & \\ \cdots & & & \\ r_{n1}\frac{Q_n}{P_n+Q_n} & \cdots & & 1 \end{bmatrix}^{-1} \times \begin{bmatrix} \frac{P_1}{P_1+Q_1} & 0 & \cdots & 0 \\ 0 & \frac{P_2}{P_2+Q_2} & \cdots & 0 \\ \cdots & & & \\ 0 & 0 & \cdots & \frac{P_n}{P_n+Q_n} \end{bmatrix}. \quad (16)$$

From Eq. 14 one derives, by the above mentioned result from the theory of the Laplace transformation,

$$\lim_{t\to\infty} \tilde{\epsilon}(t) = \lim_{p\to 0} \mathfrak{M}^{-1}(p)\mathcal{P}(p)p\tilde{x}(p) \tag{17}$$

provided the limit of the left-hand term exists.

Now consider the input $\tilde{x}(t)$ whose i-th component has a step function for its n_i-th derivative (n_i is one of the numbers 0, 1, 2, $\cdots$):

$$x_i(t) = \begin{cases} 0 & \text{for } t < 0 \\ x_i^{(n_i)} \dfrac{t^{n_i}}{n_i!} & \text{for } t > 0 \end{cases} \qquad (i = 1, 2, \cdots, n). \tag{18}$$

Then

$$px_i(p) = x_i^{(n_i)} p^{-n_i} \tag{19}$$

and a glance at (16) shows that $\tilde{\epsilon}(t)$ has a limit as t increases without limit only if $n_i \leq s_i$ ($i = 1, 2, \cdots, n$). If for any i the inequality sign holds in this relation then the steady-state error to the input $x_i(t)$ is zero, and consequently defines no error coefficient. Therefore, we assume

$$n_i = s_i \qquad (i = 1, 2, \cdots, n). \tag{20}$$

Then, by (15) and (19)

$$\lim_{p\to 0} \frac{P_i(p)}{P_i(p) + Q_i(p)} px_i(p) = K_i^{(s_i)} x_i^{(s_i)}. \tag{21}$$

From (15) it also follows that

$$\lim_{p\to 0} \frac{Q_i(p)}{P_i(p) + Q_i(p)} = \lim_{p\to 0} \left[1 - \frac{P_i(p)}{P_i(p) + Q_i(p)}\right] \tag{22}$$

$$= \begin{cases} 1 & \text{if } s_i > 0 \\ 1 - K_i^{(0)} & \text{if } s_i = 0. \end{cases}$$

To avoid troublesome notation we shall omit the case where any of the s_i vanish. This latter case is of slight importance since servos with non-vanishing static errors are of little practical interest. We finally introduce the constant n-vector

$$\vec{x}^{(s)} = (x_1^{(s_1)}, x_2^{(s_2)}, \cdots, x_n^{(s_n)}). \tag{23}$$

Then, with the assumption $s_i \geq 1$ ($i = 1, 2, \cdots, n$), Eq. 17 becomes on

substitution of (16), (21), (22)

$$\lim_{t\to\infty} \vec{\epsilon}(t) = \begin{pmatrix} 1 & r_{12} & \cdots & r_{1n} \\ r_{21} & 1 & \cdots & r_{2n} \\ \cdots & & & \\ r_{n1} & r_{n2} & \cdots & 1 \end{pmatrix}^{-1} \begin{pmatrix} K_1^{(s_1)} & 0 & \cdots & 0 \\ 0 & K_2^{(s_2)} & \cdots & 0 \\ \cdots & & & \\ 0 & 0 & \cdots & K_n^{(s_n)} \end{pmatrix} \vec{x}^{(s)}. \quad (24)$$

The matrix

$$\mathfrak{K} = \begin{pmatrix} 1 & r_{12} & \cdots & r_{1n} \\ r_{21} & 1 & \cdots & r_{2n} \\ \cdots & & & \\ r_{n1} & r_{n2} & \cdots & 1 \end{pmatrix}^{-1} \begin{pmatrix} K_1^{(s_1)} & 0 & \cdots & 0 \\ 0 & K_2^{(s_2)} & \cdots & 0 \\ \cdots & & & \\ 0 & 0 & \cdots & K_n^{(s_n)} \end{pmatrix} \quad (25)$$

is the *error coefficient matrix*. It is the product of the inverse of the "coupling matrix" and of a diagonal matrix whose diagonal elements are the error coefficients for the n uncoupled servos making up the servo system.

II. STABILITY QUESTIONS

1. *Survey*

It is well known that a system, mechanical or electrical, that performs according to a system of equations like (4) of Section I is *stable* (in the mathematical sense) if and only if all the roots of the corresponding characteristic equation lie in the left half of the complex plane. It may also be stated that the time in which such a system will approach a stationary state (if any), the so-called *delay time*, is given essentially by the reciprocal of the real part of the root with the largest real part. Several criteria are known whereby it can be decided whether all the roots of a polynomial lie in a given half-plane or not without actual knowledge of the roots; one need to mention only the Routh-Hurwitz Criterion and the Nyquist Criterion. In all these respects there are no new problems in the stability of multidimensional servo systems that do not occur with one-dimensional servos. However, there is the novel problem whether and to what degree it is possible to combine one-dimensional servos, several of which may be unstable, into a stable multidimensional servo system. Furthermore, how should the coupling coefficients and other available parameters in the individual servos be chosen, so that the resulting servo system has the shortest possible delay time. A number of such problems, each dealing with a different combination of one-dimensional servos into a multidimensional system, are taken up in the subsequent paragraphs. As a rough summary of the results contained therein, it may be stated that the servos can be combined into a stable system whenever there is a sufficient number of parameters at the designer's disposal. Physical realizability of the mathematically determined components as electrical or other networks is not discussed.

2. Two-Dimensional Systems

The characteristic equation for such a system is by Eq. 5

$$\Delta(p) \equiv \begin{vmatrix} P_1(p) + Q_1(p) & r_{12}Q_1(p) \\ r_{21}Q_2(p) & P_2(p) + Q_2(p) \end{vmatrix} = 0, \tag{26}$$

where $P_1(p)$, $P_2(p)$, $Q_1(p)$, $Q_2(p)$ are polynomials with real coefficients and r_{12}, r_{21} are real numbers. The problem to be treated is: Given a servo 1, stable or unstable, characterized by its feedback transfer function $Y_1(p) = Q_1(p)/P_1(p)$, and given a servo 2 whose transfer function $Y_2(p) = Q_2(p)/P_2(p)$ is prescribed except for the values of some parameters, how should these parameters and the coupling coefficients r_{12}, r_{21} be chosen so that the two-dimensional system whose characteristic equation is Eq. 1 is stable and its delay time is a minimum?

It should be noticed that servo 1 will be unstable or stable according to whether the polynomial $P_1(p) + Q_1(p)$ has roots in the right half of the complex plane or not. We put, therefore,

$$M_i(p) = P_i(p) + Q_i(p), \qquad (i = 1, 2, \cdots) \tag{27}$$

then Eq. 26 becomes

$$\Delta(p) \equiv M_1(p)M_2(p) - r_{12}r_{21}Q_1(p)Q_2(p) = 0. \tag{28}$$

In the subsequent cases *a*, *b*, *c* we shall refer to this equation continually.

Case a

Let $Q_1(p)$, $Q_2(p)$ be given constants and $M_1(p)$ a given quadratic polynomial and let the degree $(n - 2)$ of $M_2(p)$ be prescribed.

In this case, if p_1, p_2 denote the zeros of $M_1(p)$ the above problem reduces to the question: How should the polynomial $M_2(p) = p^{n-2} + a_1p^{n-3} + \cdots + a_{n-2}$, a_i real, and the real number k be chosen so that the maximum of the real parts of the zeros of

$$\Delta(p) = (p - p_1)(p - p_2)M_2(p) + k \tag{29}$$

is a minimum?

This minimax problem has two different solutions according to whether the given zeros p_1, p_2 of $M_1(p)$ are real or complex. To make the formulation easier we call the real part of the zero (or zeros) of $\Delta(p)$ with maximum real part the *stability zero*. A polynomial whose stability zero is negative will be called *stable*. Then we have

Theorem 1. If p_1, p_2 are fixed conjugate complex numbers then among the n-th degree polynomials of form (29) the polynomial $(p - r_0)^n$, $r_0 = \operatorname{Re} p_1 - \cot\frac{\pi}{n}|\operatorname{Im} p_1|$, has the smallest stability zero. If p_1, p_2 are real then $\frac{1}{2}(p_1 + p_2)$ is the greatest lower bound of the

stability zeros of polynomials of form (29), and this value is attained by polynomials of even degree only, unless $p_1 = p_2$.

Proof: A polynomial $\Delta(p)$ with real coefficients is of form (29) if and only if

$$\Delta(p_1) = \Delta(p_2) = \text{real}. \tag{30}$$

For, putting $\Delta(p_1) = k$, Eq. 30 implies that $\Delta(p) - k$ has the factor $(p - p_1)(p - p_2)$ with real coefficients. Hence

$$M_2(p) = \frac{\Delta(p) - \Delta(p_1)}{(p - p_1)(p - p_2)} \tag{31}$$

is then the desired polynomial of degree $n - 2$ with real coefficients.

To prove the first part of the theorem let p_1 be the zero with positive imaginary part. One observes that the point r_0 is easily constructed geometrically. It is the intersection of the x axis and the straight line through p_1 which makes the angle π/n with the x axis (see Fig. 3).

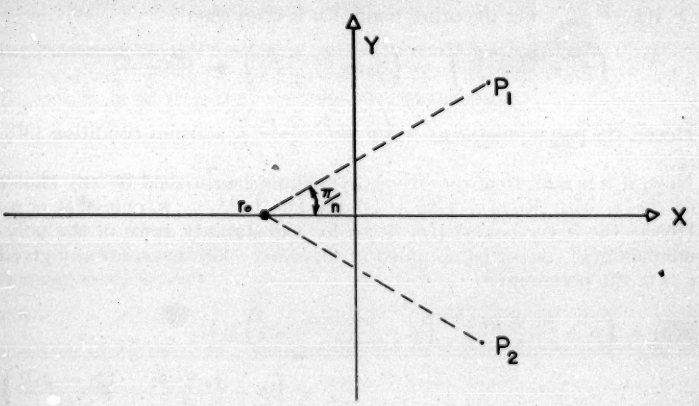

FIG. 3.

Now let $r_1, r_2, \cdots, r_n$ be the zeros of $\Delta(p)$. From (30) it follows that $(p_1 - r_1)(p_1 - r_2) \cdots (p_1 - r_n)$ is real. Therefore,

$$\sum_{i=1}^{n} \arg (p_1 - r_i) = k\pi \qquad (k = 0, \pm 1, \pm 2, \cdots). \tag{32}$$

If r is a real zero of $\Delta(p)$ and $r < r_0$ then it is immediately seen that $0 < \arg (p_1 - r) < \pi/n$. If r is a complex zero of $\Delta(p)$, then $\bar{r}$ is also a zero of $\Delta(p)$, and a simple geometric argument shows that if $\text{Re}\, r < r_0$ then $0 < \arg (p_1 - r) + \arg (p_1 - \bar{r}) < 2\pi/n$. Hence, if

$\operatorname{Re} r_i < r_0$ for $i = 1, 2, \cdots, n$ then

$$0 < \sum_{i=1}^{n} \arg (p_1 - r_i) < \pi,$$

which contradicts (32). Thus, we have shown that the stability zero of $\Delta(p)$ must be $\geq r_0$. On the other hand, it is evident from Fig. 3 that

$$(p_1 - r_0)^n = (p_2 - r_0)^n = - |p_1 - r_0|^n.$$

Hence, the polynomial $\Delta(p) = (p - p_0)^n$ satisfies condition (30). Thus, the first part of Theorem 1 is proved. Next assume p_1, p_2 real, and let again $r_1, r_2, \cdots, r_n$ be the zeros of $\Delta(p)$. Then we must have, by condition (30)

$$(p_1 - r_1)(p_1 - r_2) \cdots (p_1 - r_n) = (p_2 - r_1)(p_2 - r_2) \cdots (p_2 - r_n). \quad (33)$$

It is readily seen that the absolute value of the left-hand term of this equation is smaller than that of the right-hand term if $\operatorname{Re} r_i < \frac{1}{2}(p_1 + p_2)$ for $i = 1, 2, \cdots, n$. Hence, the stability zero of $\Delta(p)$ must be $\geq \frac{1}{2}(p_1 + p_2)$. On the other hand if n is even then

$$\left(p_1 - \frac{p_1 + p_2}{2}\right)^n = \left(p_2 - \frac{p_1 + p_2}{2}\right)^n = \left|\frac{p_1 - p_2}{2}\right|^n.$$

Hence, the polynomial $\Delta(p) = \left(p - \frac{p_1 + p_2}{2}\right)^n$ satisfies condition (30).

Now, if n is odd, then condition (33) cannot be satisfied by any choice of the r_i with $\operatorname{Re} r_i \leqq \frac{1}{2}(p_1 + p_2)$ for $i = 1, 2, \cdots, n$ unless $p_1 = p_2$. Hence, the lower bound $\frac{1}{2}(p_1 + p_2)$ for the stability zeros of the polynomials $\Delta(p)$ cannot be obtained in this case. However, for any given $\epsilon > 0$, the polynomial

$$\Delta(p) = \left(p - \frac{p_1 + p_2}{2}\right)^{n-2}\left(p - \frac{p_1 + p_2}{2} - \epsilon\right) \times \left(p - \frac{p_1 + p_2}{2} + \frac{(p_1 - p_2)^2}{4\epsilon}\right)$$

satisfies condition (30), and its stability zero is $\frac{1}{2}(p_1 + p_2) + \epsilon$. This completes the proof of Theorem 1.

It follows from Theorem 1 that the two-dimensional servo system whose characteristic polynomial is (29) can always be made stable unless

1. p_1, p_2 are conjugate complex numbers and $|\arg p_1| \leq \frac{\pi}{n}$.
2. p_1, p_2 are real and $p_1 + p_2 \geq 0$.

In the first case stability of the servo system can always be achieved with as small a time delay as desired if n is allowed to be sufficiently large. In case 2, no improvement is achieved by choice of larger n.

Case b

Let $Q_1(p)$ be a given polynomial of degree $(n-1)$ and $M_1(p)$ a given polynomial of degree n and let the degree $(n-1)$ of $Q_2(p)$ and the degree n of $M_2(p)$ be prescribed.

In this case our problem reduces to the question: How should the polynomials $M_2(p) = A_0p^n + A_1p^{n-1} + \cdots + A_n$ and $r_{12}r_{21}Q_2(p) = B_1p^{n-1} + B_2p^{n-2} + \cdots + B_n$ be chosen so that

$$\Delta(p) = M_1(p)M_2(p) - r_{12}r_{21}Q_1(p)Q_2(p) \tag{28}$$

has a stability zero as small as possible?

The result is that there exist polynomials $M_2(p)$ and $r_{12}r_{21}Q_2(p)$ which make $\Delta(p)$ any arbitrary polynomial of degree $2n$. To prove this let

$$\begin{aligned} M_1(p) &= p^n + a_1p^{n-1} + \cdots + a_n \\ Q_1(p) &= b_1p^{n-1} + b_2p^{n-2} + \cdots + b_n \end{aligned}$$

then

$$\Delta(p) = A_0p^{2n} + (a_1A_0 + A_1)p^{2n-1} + \cdots + a_nA_n - b_nB_n.$$

Let us assume that we want $\Delta(p)$ to become the polynomial $c_0p^{2n} + c_1p^{2n-1} + \cdots + c_{2n}$. Then we must choose the $(2n+1)$ variables $A_0, \cdots, A_n, B_1, \cdots, B_n$ so as to satisfy the following linear system

$$\begin{aligned} A_0 &= c_0 \\ a_1A_0 + A_1 &= c_1 \\ &\cdots \\ a_nA_n - b_nB_n &= c_{2n}. \end{aligned} \tag{34}$$

System (34) has a unique solution for $A_0, \cdots, A_n B_1, \cdots, B_n$ unless the homogeneous system

$$\begin{aligned} A_0 &= 0 \\ a_1A_0 + A_1 &= 0 \\ &\cdots \\ a_nA_n - b_nB_n &= 0 \end{aligned} \tag{35}$$

has a non-trivial solution. Any non-trivial solution of (35) must have $A_0 = A_1 = 0$. For the values $c_0 = c_1 = \cdots = c_{2n} = 0$, Eq. 28 becomes

$$\begin{aligned} \Delta(p) = {} & (p^n + a_1p^{n-1} + \cdots + a_n)(A_2p^{n-2} + \cdots + A_n) \\ & - (b_0p^{n-1} + \cdots + b_n)(B_0p^{n-1} + \cdots + B_n) = 0. \end{aligned}$$

This implies

$$\frac{b_0p^{n-1} + \cdots + b_n}{p^n + a_1p^{n-1} + \cdots + a_n} = \frac{A_2p^{n-2} + \cdots + A_n}{B_0p^{n-1} + \cdots + B_n},$$

that is,

$$\frac{b_0p^{n-1} + \cdots + b_n}{p^n + a_1p^{n-1} + \cdots + a_n} = \frac{Q_1(p)}{M_1(p)} = \frac{Q_1(p)}{Q_1(p) + P_1(p)}$$

was not in lowest terms to start with. However, since $Y_1(p) = \dfrac{Q_1(p)}{P_1(p)}$ is always assumed to be reduced to lowest terms we have a contradiction. Hence, system (35) does not have a non-trivial solution, and consequently system (34) has a unique solution for every choice of $c_0, c_1, \cdots, c_{2n}$. Therefore, no matter what $Q_1(p)$ and $M_1(p)$ are given, one can always find $M_2(p)$ and $r_{12}r_{21}Q_2(p)$ so as to make $\Delta(p)$ any arbitrary polynomial.

Case c.1

Let $M_1(p)$ be a given quadratic polynomial and $Q_1(p)$ be a given linear polynomial and let the degree 2 of $M_2(p)$ and the degree zero of $Q_2(p)$ be prescribed.

In this case the question is: For which $Q_1(p)$ and $M_1(p)$ do there exist $Q_2(p)$ and $M_2(p)$ such that $\Delta(p) = M_1(p)M_2(p) - r_{12}r_{21}Q_1(p)Q_2(p)$ is a stable polynomial?

Let

$$\begin{aligned} M_1 &= p^2 + ap + b \\ M_2 &= p^2 + Ap + B \\ Q_1 &= dp + e \\ -r_{12}r_{21}Q_2 &= E. \end{aligned}$$

The result is stated in the following.

Theorem 2. A necessary and sufficient condition for the existence of $M_2(p)$ and $Q_2(p)$ so as to make $\Delta(p)$ stable is that there should be at least one change of sign in the sequence

$$d;\ -e;\ ae - bd;\ eb - a(ae - bd);\ (a^2 - b)(ae - bd) - abe. \quad (36)$$

Proof:

$$\begin{aligned} \Delta(p) &= M_1(p)M_2(p) - r_{12}r_{21}Q_1(p)Q_2(p) \\ &= p^4 + (a + A)p^3 + (b + B + aA)p^2 \\ &\qquad + (aB + bA + dE)p + bB + eE \\ &= p^4 + c_1p^3 + c_2p^2 + c_3p + c_4. \end{aligned}$$

Since the four coefficients c_1, c_2, c_3, c_4 depend on the parameters A, B, E there must be a linear relation among the c_i. One such relation is

$$\begin{aligned} dc_4 - ec_3 + (ae - bd)c_2 + (eb - a(ae - bd))c_1 \\ + (a^2 - b)(ae - bd) - abe = 0. \end{aligned} \quad (37)$$

It is well known that in order for $\Delta(p)$ to be stable it is necessary that c_1, c_2, c_3, c_4 be positive. Hence, if Eq. 37 is to hold it is necessary that there be a change of sign in the sequence (36).

To see the sufficiency of the condition choose the c_i as the coefficients of the following stable polynomial:

$$\Delta(p) = [(p + h)^2 + k^2]^2 \qquad (h > 0),$$

that is,

$$c_1 = 4h, \quad c_2 = 6h^2 + 2k^2, \quad c_3 = 4h^3 + 24k^2h, \quad c_4 = h^4 + 2k^2h^2 + k^4.$$

Then Eq. 37 becomes

$$dh^4 - 4eh^3 + [2dk^2 + 6(ae - bd)]h^2 + 4(abd + eb - a^2e - 6ek^2)h + dk^4 + 2k^2(ae - bd) + (a^2 - b)(ae - bd) - abe = 0. \quad (38)$$

If $(a^2 - b)(ae - bd) - abe$ is different in sign from d then for $k = 0$ there exists a positive h satisfying (38). If $(a^2 - b)(ae - bd) - abe$ and d have the same sign then the term in (38) free of h, which is a biquadratic in k, will be different in sign from d for some real k if

$$(ae - bd)^2 - d[(a^2 - b)(ae - bd) - abe] > 0$$

and

$$(ae - bd)d < 0.$$

Since the first condition can be written as $a^2e^2 - a^2d(ae - bd) > 0$, the two conditions will be satisfied if $(ae - bd)$ and d do not have the same sign. Under this condition then the term in (38) free of h will be of different sign from d for some real k. For this k there will exist a positive h satisfying (38).

If d, $ae - bd$, and $(a^2 - b)(ae - bd) - abe$ have the same sign, then choose the c_i as the coefficients of the polynomial $p^3(p + h)$, that is, $c_1 = h$, $c_2 = c_3 = c_4 = 0$. Then Eq. 37 becomes

$$[eb - a(ae - bd)]h + (a^2 - b)(ae - bd) - abe = 0. \quad (39)$$

If $eb - a(be - bd)$ and $(a^2 - b)(ae - bd) - abe$ are not of the same sign, there exists a positive h satisfying (39). The polynomial $p^3(p + h)$ has zero for its stability zero. However $(p + \epsilon)^3(p + h)$, for $\epsilon > 0$, has a negative stability zero and if the coefficients of $p^3(p + h)$ for some $h > 0$ satisfy (37) then the coefficients of $(p + \epsilon)^3(p + h)$ satisfy (37) for sufficiently small $\epsilon > 0$ and some $h > 0$.

If d, $ae - bd$, $(a^2 - b)(ae - bd) - abe$, and $eb - a(ae - bd)$ are all of the same sign, then choose the c_i as coefficients of the polynomial $p(p + h)^3$, that is, $c_1 = 3h$, $c_2 = 3h^2$, $c_3 = h^3$, $c_4 = 0$. Then Eq. 37 becomes

$$-eh^3 + 3h^2(ae - bd) + 3h(abd + eb - a^2e) + (a^2 - b)(ae - bd) - abe = 0. \quad (40)$$

If $-e$ and $(a^2 - b)(ae - bd) - abe$ are not of the same sign, there exists a positive h satisfying (40). The polynomial $p(p + h)^3$ has zero for its stability zero, but as above it may be replaced by a stable polynomial $(p + \epsilon)(p + h)^3$.

Thus, the condition that there be a sign change in sequence (37) is also proved to be sufficient.

Case c.2

Let $Q_1(p)$ be a given linear polynomial and $M_1(p)$ be a given cubic polynomial and let the degree 1 of $Q_2(p)$ and the degree 3 of $M_2(p)$ be prescribed.

In this case the question is: For which $Q_1(p)$ and $M_1(p)$ do there exist $Q_2(p)$ and $M_2(p)$ such that $\Delta(p) = M_1(p)M_2(p) - r_{12}r_{21}Q_1(p)Q_2(p)$ is a stable polynomial?

Let

$$\begin{aligned} M_1(p) &= p^3 + bp^2 + cp + d \\ M_2(p) &= p^3 + Bp^2 + Cp + D \\ Q_1(p) &= ap + e \\ -r_{12}r_{21}Q_2(p) &= Ap + E. \end{aligned}$$

The result is stated in the following

Theorem 3. A necessary and sufficient condition for the existence of $M_2(p)$ and $Q_2(p)$ so as to make $\Delta(p)$ stable is that at least one of the following numbers should be negative:

$$\begin{aligned} &-ae, \quad ace - a^2d - be^2 = L_1, \quad -bL_1 + ade - ce^2 = L_2, \\ &-bL_2 - cL_1 + de^2 = L_3, \quad -bL_3 - cL_2 - dL_1 = L_4. \end{aligned} \tag{41}$$

Proof:

$$\begin{aligned} \Delta(p) &= M_1(p)M_2(p) - r_{12}r_{21}Q_1(p)Q_2(p) \\ &= p^6 + (b + B)p^5 + (c + C + bB)p^4 + (d + D + bC + Bc)p^3 \\ &\quad + (bD + dB + cC + aA)p^2 + (cD + dC + aE + Ae)p \\ &\quad + dD + eE \\ &= p^6 + c_1p^5 + c_2p^4 + c_3p^3 + c_4p^2 + c_5p + c_6. \end{aligned}$$

Since the six coefficients $c_1, c_2, c_3, c_4, c_5, c_6$ depend on the five parameters A, B, C, D, E, there must be a linear relation among the c_i. One such relation is:

$$a^2c_6 - aec_5 + e^2c_4 + L_1c_3 + L_2c_2 + L_3c_1 + L_4 = 0. \tag{42}$$

Since, in order for $\Delta(p)$ to be stable it is necessary that all its coefficients c_i be positive, there must be a negative element in the sequence (41).

To see the sufficiency of the condition, choose the c_i as coefficients of the polynomial $(p + h)^6$, that is, $c_1 = 6h$, $c_2 = 15h^2$, $c_3 = 20h^3$, $c_4 = 15h^4$, $c_5 = bh^5$, $c_6 = h^6$. Then Eq. 42 becomes

$$a^2h^6 - 6aeh^5 + 15e^2h^4 + 20L_1h^3 + 15L_2h^2 + 6L_3h + L_4 = 0. \tag{43}$$

If $L_4 < 0$ then there is a positive h satisfying (43).

If $L_4 > 0$, then choose the c_i as coefficients of the polynomial $p^5(p + h)$, that is, $c_1 = h$, $c_2 = c_3 = c_4 = c_5 = c_6 = 0$. Then Eq. 42 becomes

$$L_3h + L_4 = 0. \tag{44}$$

If $L_3 < 0$ then, since we assumed $L_4 > 0$, there exists a positive h satisfying (44). As above, the polynomial $p^5(p + h)$ may be replaced by a stable polynomial $(p + \epsilon)^5(p + h)$.

If $L_3 > 0$, $L_4 > 0$, then choose the c_i as the coefficients of the polynomial $p^4(p + h)^2$, that is, $c_1 = 2h$, $c_2 = h^2$, $c_3 = c_4 = c_5 = c_6 = 0$. Then Eq. 42 becomes

$$h^2L_2 + 2hL_3 + L_4 = 0. \tag{45}$$

If $L_2 < 0$ then, since we assumed $L_4 > 0$, there exists a positive h satisfying (45). Again the polynomial $p^4(p + h)$ may be replaced by a stable polynomial $(p + \epsilon)^4(p + h)^2$.

If $L_2 > 0$, $L_3 > 0$, $L_4 > 0$, then choose the c_i as the coefficients of the polynomial $p^3(p + h)^3$, that is, $c_1 = 3h$, $c_2 = 3h^2$, $c_3 = h^3$, $c_4 = c_5 = c_6 = 0$. Then Eq. 42 becomes

$$h^3L_1 + 3h^2L_2 + 3hL_3 + L_4 = 0. \tag{46}$$

If $L_1 < 0$, then, since we assumed $L_4 > 0$, there exists a positive h satisfying (46). Again the polynomial $p^3(p + h)^3$ may be replaced by a stable polynomial $(p + \epsilon)^3(p + h)^3$.

If $L_1 > 0$, $L_2 > 0$, $L_3 > 0$, $L_4 > 0$, then choose the c_i as the coefficients of the polynomial $p(p + h)^5$, that is, $c_1 = 5h$, $c_2 = 10h^2$, $c_3 = 10h^3$, $c_4 = 5h^4$, $c_5 = h^5$, $c_6 = 0$. Then Eq. 42 becomes

$$-aeh^5 + 5e^2h^4 + 10h^3L_1 + 10h^2L_2 + 5hL_3 + L_4 = 0. \tag{47}$$

If $-ae < 0$ then, since we assumed $L_4 > 0$, there exists a positive h satisfying (47). As before, the polynomial $p(p + h)^5$ may be replaced by a stable polynomial $(p + \epsilon)(p + h)^5$.

Thus, Theorem 3 is completely proved. It should be noted that $\Delta(p)$ can be made stable if $ae > 0$. This latter condition holds when $Q_1(p)$ is stable. Hence, if $Q_1(p)$ is stable, then $\Delta(p)$ can be made stable.

3. N-Dimensional Systems

The characteristic equation for such a system is by Eq. 5

$$\Delta(p) \equiv \begin{vmatrix} P_1(p) + Q_1(p) & r_{12}Q_1(p) & \cdots & r_{1n}Q_1(p) \\ r_{21}Q_2(p) & P_2(p) + Q_2(p) & \cdots & r_{2n}Q_2(p) \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ r_{n1}Q_n(p) & r_{n2}Q_n(p) & \cdots & P_n(p) + Q_n(p) \end{vmatrix} = 0, \tag{48}$$

where the $P_i(p)$, $Q_i(p)$ are polynomials with real coefficients and r_{ij} are real numbers. The problem to be treated in this paragraph is: Given servos $1, 2, \cdots, n-1$ stable or unstable, characterized by their feedback transfer functions $Y_i(p) = \dfrac{Q_i(p)}{P_i(p)}$, and given a servo n whose

transfer function $Y_n(p) = Q_n(p)/P_n(p)$ is prescribed except for the values of some parameters, how should these parameters and the coupling coefficients r_{ij} be chosen so that the n-dimensional system whose characteristic equation is Eq. 48 is stable? As before, we use the notation, $M_i(p) = P_i(p) + Q_i(p)$.

Case a

Let $Q_1(p), \cdots, Q_n(p)$ be given constants and $M_1(p), \cdots, M_{n-1}(p)$ given quadratic polynomials and let the degree 2 of $M_n(p)$ be prescribed.

In this case the problem reduces to the question: Under what restrictions on $M_1, \cdots, M_{n-1}$ can M_n, r_{ij} be chosen so that $\Delta(p)$ is stable? Let

$$M_1 M_2 \cdots M_{n-1} = p^{2n-2} + a_1 p^{2n-3} + a_2 p^{2n-4} + a_3 p^{2n-5} + \cdots + a_{2n-2} \quad (49)$$

$$M_n = p^2 + Ap + B.$$

Then

$$\Delta(p) = (p^{2n-2} + a_1 p^{2n-3} + a_2 p^{2n-4} + a_3 p^{2n-5} + \cdots + a_{2n-2})(p^2 + Ap + B) + ((p^{2n-4}))$$

$$= p^{2n} + c_1 p^{2n-1} + c_2 p^{2n-2} + c_3 p^{2n-3} + c_4 p^{2n-4} + \cdots + c_{2n}.$$

The result is stated in the following

Theorem 4. A necessary and sufficient condition for the existence of $M_n(p)$ and r_{ij} to make $\Delta(p)$ stable is that at least one of the numbers

$$-a_1; \quad a_1^2 - a_2; \quad 2a_1a_2 - a_1^3 - a_3 \quad (50)$$

be negative.

Proof: Let us assume that the coefficients $c_4, c_5, \cdots, c_{2n}$ may be given any real values by proper choice of the r_{ij} for any given A, B. Since the three coefficients c_1, c_2, c_3 depend on only the two parameters A, B, there must be a linear relation among c_1, c_2, c_3. One such relation is:

$$c_3 - a_1c_2 + (a_1^2 - a_2)c_1 + 2a_1a_2 - a_1^3 - a_3 = 0. \quad (51)$$

Since in order for $\Delta(p)$ to be stable it is necessary that c_1, c_2, c_3 are positive there must be a negative term in sequence (50).

To see the sufficiency of the condition, choose the c_i as the coefficients of the polynomial $(p + h)^{2n}$, that is, $c_1 = 2nh$, $c_2 = \binom{2n}{2}h^2$, $c_3 = \binom{2n}{3}h^3$. Then Eq. 51 becomes

$$\binom{2n}{3}h^3 - \binom{2n}{2}a_1h^2 + 2n(a_1^2 - a_2)h + 2a_1a_2 - a_1^3 - a_3 = 0. \quad (52)$$

If $2a_1a_2 - a_1^3 - a_3 < 0$, then there exists a positive h satisfying (52).

If $2a_1a_2 - a_1^3 - a_3 > 0$, choose the c_i as the coefficients of the polynomial $p^{2n-1}(p + h)$, that is, $c_1 = h$, $c_2 = c_3 = 0$. Then Eq. 51 becomes

$$(a_1^2 - a_2)h + 2a_1a_2 - a_1^3 - a_3 = 0. \tag{53}$$

If $a_1^2 - a_2 < 0$, there exists a positive h satisfying (53). As was done before, the polynomial $p^{2n-1}(p + h)$ may be replaced by a stable polynomial $(p + \epsilon)^{2n-1}(p + h)$.

If $2a_1a_2 - a_1^3 - a_3 > 0$, $a_1^2 - a_2 > 0$, choose the c_i as the coefficients of the polynomial $p^{2n-2}(p + h)^2$, that is, $c_1 = 2h$, $c_2 = h^2$, $c_3 = 0$. Then Eq. 51 becomes

$$-a_1h^2 + 2(a_1^2 - a_2)h + 2a_1a_2 - a_1^3 - a_3 = 0. \tag{54}$$

If $-a_1 < 0$, there exists a positive h satisfying (54). Replacing $p^{2n-2}(p + h)$ by a stable $(p + \epsilon)^{2n-2}(p + h)^2$ is done as before. Thus, Theorem 4 has been proved on the assumption that the coefficients $c_4, \cdots, c_{2n}$ can be made to assume any real values. We have $n^2 - n$ parameters at our disposal to adjust the $2n - 3$ coefficients $c_4, \cdots, c_{2n}$. Hence, except for very special exceptions the assumption is justified.

An interesting special case is

$$M_1(p) = M_2(p) = \cdots = M_{n-1}(p) = p^2 + ap + b.$$

Then, by (49),

$$a_1 = (n - 1)a, \qquad a_2 = \binom{n-1}{2}a^2 + (n - 1)b,$$

$$a_3 = 2\binom{n-1}{2}ab + \binom{n-1}{3}a^3.$$

The necessary and sufficient condition of Theorem 4 excludes the following region:

$$a < 0, \qquad b < \frac{n+1}{6}a^2.$$

If the zeros of $M_1(p), \cdots, M_{n-1}(p)$ are $\alpha \pm i\beta$, the excluded values are:

$$\alpha > 0, \qquad \beta^2 < \frac{2n-1}{3}\alpha^2.$$

Hence, if $M_1(p) = M_2(p) = \cdots = M_{n-1}(p)$ have conjugate complex zeros in the region $|\arg z| \leq \arctan\sqrt{\frac{2n-1}{3}}$ then no $M_n(p)$ can be found that makes $\Delta(p)$ stable.

If the zeros of $M_1(p) = \cdots = M_{n-1}(p)$ are real, say α_1, α_2, then the only excluded region is $\alpha_1 + \alpha_2 > 0$. In every other case an $M_n(p)$ can be found so as to make $\Delta(p)$ stable.

Case b

Let $Q_1(p), Q_2(p), \cdots, Q_{n-1}(p)$ be given constants and $M_1(p), \cdots, M_{n-1}(p)$ given quadratic polynomials such that the condition of Case *a* precludes the possibility of using a quadratic $M_n(p)$ to stabilize $\Delta(p)$.

The question is: can a cubic polynomial $M_n(p)$ be found so as to make $\Delta(p)$ stable?

That this can actually be done except in some very exceptional cases is seen as follows. Let

$$\begin{aligned} M_1 M_2 \cdots M_{n-1} &= p^{2n-2} + a_1 p^{2n-3} + \cdots + a_{2n-2} \\ M_3 &= p^3 + Ap^2 + Bp + C. \end{aligned} \tag{55}$$

Then

$$\begin{aligned} \Delta(p) &= (p^{2n-2} + a_1 p^{2n-3} + \cdots + a_{2n-2})(p^3 + Ap^2 + Bp + C) \\ &\qquad + ((p^{2n-3})) \\ &= p^{2n+1} + c_1 p^{2n} + c_2 p^{2n-1} + c_3 p^{2n-2} + \cdots + c_{2n+1}, \end{aligned} \tag{56}$$

where

$$\begin{aligned} c_1 &= A + a_1 \\ c_2 &= a_1 A + B + a_2 \\ c_3 &= a_2 A + a_1 B + C + a_3. \end{aligned} \tag{57}$$

System (57) can, for arbitrary given c_1, c_2, c_3, be solved for the unknowns A, B, C. Comparison of the coefficients of the remaining $(2n-2)$ powers $p^{2n-3}, p^{2n-4}, \cdots, p^0$ in Eq. 56 would lead to $(2n-2)$ equations involving the $n^2 - n$ adjustable parameters r_{ij}. These equations can, except for very special cases, be solved no matter what the right-hand terms $c_4, c_5, \cdots, c_{2n+1}$ are. Hence, $\Delta(p)$ can by choice of A, B, C, and of the parameters r_{ij} be made any polynomial of degree $(2n+1)$, in particular a stable polynomial.

III. RMS-ERROR CRITERION FOR MULTIDIMENSIONAL SERVO SYSTEMS

1. Preliminary Discussion

In order to determine what is the best possible servo system of a specified type to meet a given need Hall[3] and Phillips[4] have independently proposed and developed the criterion that the *rms error* of the output should be minimized. To apply this criterion it is necessary not only to specify the class of servos among which the optimal servo is to be determined but also the occurring inputs or, at least, their frequency

[3] A. C. Hall, "The Analysis and Synthesis of Linear Servomechanisms," Cambridge, Technology Press, 1943.

[4] R. S. Phillips, "Servomechanisms," Radiation Laboratory Report No. 372, May, 1943; also Chapter VII of the treatise by James et al. (footnote 2). Much of the basic theory was developed by N. Wiener in "The Extrapolation, Interpolation and Smoothing of Stationary Time Series," NDRC Report, 1942. See also N. Wiener, "The Extrapolation, Interpolation, and Smoothing of Stationary Time Series," Cambridge, Technology Press, 1949.

characteristics, more precisely the *spectral density* of the inputs. If transient loading and random disturbances (noise) have an appreciable effect on the performance of the servo and if their spectral densities (or auto-correlation functions) are known, these should be included among the inputs. It is clear that these inputs should not be faithfully reproduced, hence they must be treated differently from the inputs which are signals.

In this section an adaptation of the rms-error criterion to the case of multidimensional servo systems is developed. The main result of this investigation is that the methods for minimizing the rms-error that have been developed for one-dimensional servos can readily be extended to apply for multidimensional servo systems.

2. Calculation of the RMS-Error

As in Section I, let $x_i(t)$ $(i = 1, 2, \cdots, n)$ denote the signal input functions, $y_i(t)$ the signal output functions, and $\epsilon_i(t) = x_i(t) - y_i(t)$ the corresponding error functions of the n-dimensional servo system. Then, by Eq. 8 we have formally

$$\epsilon_i(t) = \sum_{j=1}^{n} R_{ij}(D)x_j(t), \quad D = \frac{d}{dt} \qquad (i = 1, 2, \cdots, n), \tag{58}$$

where the $R_{ij}(p)$, which are rational functions of a complex variable p, are the components of the loop impedance matrix $\mathfrak{R}(p) = \mathfrak{M}^{-1}(p)\mathfrak{P}(p)$.

The *time average* $\tilde{u}$ of a time-dependent variable $u(t)$ is defined as

$$\tilde{u} = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} u(t)\,dt. \tag{59}$$

The *spectral density* $U(f)$ of the function $u(t)$ is defined as

$$U(f) = \lim_{T\to\infty} \frac{1}{T}\left|\int_{-T}^{T} u(t)\,e^{-2\pi i f t}\,dt\right|^2. \tag{60}$$

It is a well known result of general harmonic analysis[5] that

$$\tilde{u}^2 = \int_0^\infty U(f)\,df. \tag{61}$$

The *total rms-error* for the time-dependent errors $\epsilon_1(t), \epsilon_2(t), \cdots, \epsilon_n(t)$ is defined as the square root of the time average of $\sum_{i=1}^{n} \epsilon_i^2(t)$, that is,

$$\text{rms-error} = \sqrt{\tilde{\epsilon}^2} = \lim_{T\to\infty}\left[\frac{1}{2T}\int_{-T}^{T}\sum_{i=1}^{n}\epsilon_i^2(t)\,dt\right]^{\frac{1}{2}}. \tag{62}$$

[5] See N. Wiener, "Generalized Harmonic Analysis," *Acta Mathematica*, Vol. 55, pp. 116–258 (1930).

By Eq. 61, the rms-error can also be obtained from

$$\bar{\epsilon}^2 = \int_0^\infty E(f)\, df, \tag{63}$$

where $E(f)$ is the *total spectral density of the errors* $e_i(t)$, that is,

$$E(f) = \sum_{i=1}^{n} E_i(f) = \sum_{i=1}^{n} \lim_{T\to\infty} \frac{1}{T}\left| \int_{-T}^{T} \epsilon_i(t)\, e^{-2\pi i f t}\, dt \right|^2. \tag{64}$$

We also need the *spectral density matrix* $\mathfrak{X}(f)$ *of the inputs* $x_i(t)$, whose elements $X_{ij}(f)$ are defined by

$$X_{ij}(f) = \lim_{T\to\infty} \frac{1}{T} \int_{-T}^{T} x_i(t)\, e^{-2\pi i f t}\, dt \int_{-T}^{T} x_j(t)\, e^{2\pi i f t}\, dt. \tag{65}$$

It is seen that $X_{ij}(f)$ is the conjugate complex of $X_{ji}(f)$. Therefore, the matrix $\mathfrak{X}(f)$ is Hermitian. It is also positive semidefinite since for any complex numbers $u_1, u_2, \cdots, u_n$

$$\sum_{i,j=1}^{n} X_{ij}(f) u_i \bar{u}_j = \lim_{T\to\infty} \frac{1}{T}\left| \sum_{i=1}^{n} u_i \int_{-T}^{T} x_i(t)\, e^{-2\pi i f t}\, dt \right|^2 \geq 0.$$

The input functions $x_i(t)$ and error functions $\epsilon_i(t)$ are related to each other by Eq. 58. From this follows a relationship between the spectral density matrix of the inputs and the spectral densities of the errors. By a result of general harmonic analysis[6] one deduces from Eq. 58

$$E_i(f) = \sum_{j,k=1}^{n} R_{ik}(2\pi i f)\overline{R_{ij}(2\pi i f)} X_{kj}(f). \tag{66}$$

In the following we will often omit the argument $2\pi i f$ from the functions $R_{ij}(2\pi i f)$ and the argument f from the functions $X_{ij}(f)$. From (66) follows

$$E(f) = \sum_{i,j,k=1}^{n} R_{ik}\overline{R_{ij}} X_{kj}. \tag{67}$$

Using the notation a^* for the conjugate transpose of a matrix a, and $\operatorname{tr} a$ for the trace of matrix a, that is, the sum of terms along the principal diagonal, Eq. 67 can be rewritten as

$$E(f) = \operatorname{tr}(\mathfrak{R}\mathfrak{R}^*\mathfrak{X}). \tag{68}$$

Finally, by Eq. 63,

$$\bar{\epsilon}^2 = \int_0^\infty \operatorname{tr}(\mathfrak{R}\mathfrak{R}^*\mathfrak{X})\, df. \tag{69}$$

[6] See the works by N. Wiener, cited in footnotes 4 and 5; also, Section 6.9 of the treatise by James et al., footnote 2.

Thus, the mean square error is expressed in terms of the loop impedance matrix of the servo system and the spectral density matrix of the inputs.

3. Transformation to Principal Axes

To make the methods developed for one-dimensional servos available to the present case of multidimensional systems it is desirable to have $\bar{\epsilon}^2$ expressed as a sum of terms as they occur in the one-dimensional case. There the impedance matrix $\mathfrak{R}(2\pi if)$ and the spectral density matrix $\mathfrak{X}(f)$ consist of a single term $R(2\pi if)$, $X(f)$, respectively, and Eq. 69 becomes

$$\bar{\epsilon}^2 = \int_0^\infty |R|^2 X df. \tag{70}$$

To obtain the rms-error for an n-dimensional servo system as a sum of terms like (70) one may proceed in two different ways. Depending on the special case at hand, one or the other way may be preferable. First, since $\mathfrak{X}$ is Hermitian there exists a unitary matrix $\mathfrak{U}$, $\mathfrak{U}\mathfrak{U}^* = \mathcal{E}$ (the identity matrix), such that the transformed matrix $\mathfrak{U}\mathfrak{X}\mathfrak{U}^*$ is a diagonal matrix, say

$$\mathfrak{U}\mathfrak{X}\mathfrak{U}^* = \text{diag}\,(X_1, X_2, \cdots, X_n). \tag{71}$$

The X_i, the characteristic values of the matrix , are real and non-negative since $\mathfrak{X}$ is positive semidefinite. Writing $\mathfrak{R}' = ((R'_{ij}))$ for the cogrediently transformed matrix $\mathfrak{U}\mathfrak{R}\mathfrak{U}^*$, Eq. 69 becomes, since the trace of a matrix is a unitary invariant,

$$\begin{aligned}\bar{\epsilon}^2 &= \int_0^\infty \text{tr}\,(\mathfrak{U}\mathfrak{R}\mathfrak{U}^*\mathfrak{U}\mathfrak{R}^*\mathfrak{U}^*\mathfrak{U}\mathfrak{X}\mathfrak{U}^*)\,df \\ &= \sum_{i,j=1}^{n} \int_0^\infty |R'_{ij}|^2 X_i\,df.\end{aligned} \tag{72}$$

This is a sum of non-negative terms each of the form (70). The method by which this result was established may be called *transformation to principal input axes.*

Second, since $\mathfrak{R}\mathfrak{R}^*$ is a Hermitian (and positive semidefinite matrix) one can find a unitary matrix $\mathfrak{V}$ such that the transformed matrix $\mathfrak{V}\mathfrak{R}\mathfrak{R}^*\mathfrak{V}^*$ is diagonal, say

$$\mathfrak{V}\mathfrak{R}\mathfrak{R}^*\mathfrak{V}^* = \text{diag}\,(R_1, R_2, \cdots, R_n), \tag{73}$$

where the R_i are the characteristic values of the matrix $\mathfrak{R}\mathfrak{R}^*$ and are real and non-negative. Writing $\mathfrak{X}' = ((X'_{ij}))$ for the cogrediently transformed matrix $\mathfrak{V}\mathfrak{X}\mathfrak{V}^*$ Eq. 69 becomes

$$\begin{aligned}\bar{\epsilon}^2 &= \int_0^\infty \text{tr}\,(\mathfrak{V}\mathfrak{R}\mathfrak{R}^*\mathfrak{V}^*\mathfrak{V}\mathfrak{X}\mathfrak{V}^*)\,df \\ &= \sum_{i=1}^{n} \int_0^\infty R_i X'_{ii}\,df.\end{aligned} \tag{74}$$

Again we have a sum of terms each of the form (70). We can show that the X'_{ii} are real and non-negative. For, letting V_{ij} denote the components of $\mathfrak{V}$ we have

$$X'_{ii} = \sum_{j,k=1}^{n} V_{ik} X_{kj} \overline{V_{ij}}$$

and using Eq. 65 this becomes

$$X'_{ii} = \lim_{T\to\infty} \frac{1}{T} \sum_{j,k=1}^{n} V_{ik}\overline{V_{ij}} \int_{-T}^{T} x_k(t)\, e^{-2\pi i f t}\, dt \int_{-T}^{T} x_j(t)\, e^{2\pi i f t}\, dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \left| \sum_{k=1}^{n} V_{ik} \int_{-T}^{T} x_k(t)\, e^{-2\pi i f t}\, dt \right|^2 \geq 0.$$

The procedure by which expression (74) for the rms-error is obtained should, for obvious reasons, be called *transformation to principal impedance axes.*

4. The Minimization Problem

By the rms criterion, that servo system is best among a given class of servo systems which minimizes $\bar{\epsilon}^2$. Mathematically this means to minimize, for a given spectral density matrix of inputs $\mathfrak{X}(f)$, the expression (69) by choosing optimal values for the finite number of adjustable parameters in the impedance matrix $\mathfrak{R}$. This minimum problem for a function of a finite number of variables is in no way different from the corresponding problem for one-dimensional servos. Theoretically, there is no advantage in replacing expression (69) for the rms-error by one of the equivalent expressions (72) or (74), which were derived by transformations to principal axes. From a practical point of view, however, there is a significant advantage. For all but the simplest servo systems the determination of the theoretically optimal values that minimize the rms-error is a formidable proposition, and this is no less true for one-dimensional servos. The practical designer will, therefore, resort to approximation methods. From expression (70) for the rms-error (in the case of one-dimensional servos) it is evident that the problem is to make the impedance function $R(2\pi if)$ as small as possible, especially in those ranges of the frequency f in which $X(f)$ is relatively large. The practical designer carries out this program usually by the aid of attenuation phase diagrams of one of the impedance or transfer functions for the servo.

For multidimensional systems it appears from expression (74), which gives the rms-error expressed along the principal axes of the impedance matrix, that the corresponding problem is to make the n impedance functions $R_i(2\pi if)$ (the characteristic values of the loop impedance

matrix) as small as possible, especially in those ranges of the frequency f in which the spectral density components of the inputs $X'_{ii}(f)$ are relatively large. To carry out this program the practical designer may have to work with the n attenuation phase diagrams of the impedance functions $R_i(2\pi if)$ at the same time, selecting some compromise course in design which best suits the requirements of the particular problem at hand.

A similar program can be based on expression (74), which gives the rms-error expanded along the principal input axes rather than along the principal impedance axes.

5. Vector Input of Random Direction

A simple expression for the rms-error is obtained, without transformation to principal axes, in the case when the input is an n-dimensional vector $\vec{x}(t)$ whose magnitude has the given spectral density $X(f)$ and whose direction is random covering equal parts of the angular space with equal probability. The servo system is to reproduce the components of this vector along n orthogonal axes $A_1, A_2, \cdots, A_n$.

If $u_1(P), u_2(P), \cdots, u_n(P)$ denote the coordinates of a point P on the unit sphere in the coordinate system whose axes are $A_1, A_2, \cdots, A_n$, then the spectral density matrix of the input vector in the direction from the origin to the point P is given by

$$X_{ij}(f; P) = X(f)u_i(P)u_j(P). \tag{75}$$

Since we assume the point P to be a random variable which covers equal surface areas of the unit sphere with equal likelihood and since the mean value of the product $u_i(P)u_j(P)$ over the unit sphere is 0 if $i \neq j$, and is 1 if $i = j$, it follows from (75) that

$$\mathfrak{X}(f) = X(f)\mathcal{E}. \tag{76}$$

On substitution in Eq. 69 one has

$$\tilde{\epsilon}^2 = \int_0^\infty \text{tr}\,(\mathfrak{R}\mathfrak{R}^*)X\,df. \tag{77}$$

Hence, in this case the rms-error is expressed just like that for a one-dimensional servo (see Eq. 70) except that the impedance function $|R|^2$ is replaced by the impedance function tr $(\mathfrak{R}\mathfrak{R}^*)$.

It should be noted that Eq. 77 was derived by taking an *ensemble average* of the directions of the input vector rather than a time average. As is well known, the two averages are equal with a probability of 1 provided the ergodic hypothesis holds for the random process by which the input vector $\vec{x}(t)$ changes its direction.

e. The Integrated-Square Error

The total *integrated-square error* for the time-dependent errors $\epsilon_1(t), \epsilon_2(t), \cdots, \epsilon_n(t)$ is defined as

$$[\epsilon^2] = \int_{-\infty}^{\infty} \sum_{i=1}^{n} \epsilon_i^2(t)\, dt. \tag{78}$$

Comparison with Eq. 62 shows that $[\epsilon^2]$ is infinite when the rms-error $\tilde{\epsilon}^2$ is finite and positive, and $\tilde{\epsilon}^2$ is zero when $[\epsilon^2]$ is finite. The two types of errors are of a different order of magnitude and never apply in the same situation. The integrated-square error is used to measure the transient response of a stable servo system to inputs that start at a definite time, say $t = 0$, where the rms-error would only give zero.

In the same way as formula (69) was derived for the rms-error, one derives the following formula for the integrated-square error.

$$[\epsilon^2] = \int_0^{\infty} \operatorname{tr}\,(\mathfrak{R}\mathfrak{R}^* {}'\mathfrak{X})\, df, \tag{79}$$

where ${}'\mathfrak{X}(f)$ is the matrix whose elements are

$${}'X_{ij}(f) = \xi_i(2\pi i f)\overline{\xi_j(2\pi i f)} \tag{80}$$

and $\xi_i(p)$ is the analytic function an element of which is defined by the Laplace transform of $x_i(t)$,

$$\xi_i(p) = \int_0^{\infty} e^{-pt} x_i(t)\, dt. \tag{81}$$

With the replacement of the $X_{ij}(f)$ by the ${}'X_{ij}(f)$, the principal axes representations (72) and (74) also hold for $[\epsilon^2]$.

The transient response of a servo is usually tested by the use of a step-function as input. If we let the n input functions $x_1(t), \cdots, x_n(t)$ to the n-dimensional servo system be step-functions, all of them having their discontinuity at time $t = 0$,

$$\begin{aligned} x_i(t) &= 0 \text{ for } t < 0 \\ &= x_i \text{ for } t > 0 \end{aligned} \tag{82}$$

then, by (81), $\xi_i(p) = \dfrac{1}{p} x_i$ and the matrix ${}'\mathfrak{X}$ has the elements ${}'X_{ij}(f) = \dfrac{1}{4\pi^2 f^2} x_i x_j$. If one, moreover, assumes that the x_i are the components of a unit vector whose direction is random covering equal parts of the angular space with equal probability, then one obtains, similarly as in Section III.5

$$[\epsilon^2] = \frac{1}{4\pi^2} \int_0^{\infty} \frac{\operatorname{tr}\,(\mathfrak{R}\mathfrak{R}^*)}{f^2}\, df. \tag{83}$$

This error is a function only of the variable parameters of the servo system and may be used as a convenient figure of merit for the transient response of the servo system. Its evaluation is not difficult, especially when use is made of the table of integrals which was especially prepared for integrals of type (83).[7] Minimization of the function thus obtained is, of course, a more difficult matter.

7. An Example

As an example for the calculation of the integrated-square error for a step-function vector input of random direction, the two-dimensional servo system of Fig. 4 is considered. The output-error equations for this system are

$$\begin{aligned}(I_1D^2 + R_1D)y_1 &= C_1(\epsilon_1 + r_{12}\epsilon_2)\\ (I_2D^2 + R_2D)y_2 &= C_2(r_{21}\epsilon_1 + \epsilon_2),\end{aligned} \tag{84}$$

where I_i is the moment of inertia of servo motor i, R_i is the internal resistance of servo motor i, and C_i is the conversion factor that relates the output of motor i to the input to amplifier i ($i = 1, 2$). The matrices $\mathcal{P}(p)$, $\mathfrak{M}(p)$, $\mathcal{R}(p)$, of Section I.2 become

$$\begin{aligned}\mathcal{P}(p) &= \begin{pmatrix} I_1p^2+R_1p & 0 \\ 0 & I_2p^2+R_2p \end{pmatrix}\\ \mathfrak{M}(p) &= \begin{pmatrix} I_1p^2+R_1p+C_1 & r_{12}C_1 \\ r_{21}C_2 & I_2p^2+R_2p+C_2 \end{pmatrix}\\ \mathcal{R}(p) &= \frac{1}{\Delta(p)} \\ &\times \begin{pmatrix} (I_2p^2+R_2p+C_2)(I_1p^2+R_1p) & -r_{12}C_1(I_2p^2+R_2p) \\ -r_{21}C_2(I_1p^2+R_1p) & (I_1p^2+R_1p+C_1)(I_2p^2+R_2p) \end{pmatrix},\end{aligned} \tag{85}$$

where $\Delta(p)$ is the characteristic polynomial

$$\Delta(p) = (I_1p^2 + R_1p + C_1)(I_2p^2 + R_2p + C_2) - r_{12}r_{21}C_1C_2. \tag{86}$$

One easily calculates

$$\begin{aligned}\frac{\operatorname{tr}(\mathcal{R}\mathcal{R}^*)}{p^2} = \frac{1}{|\Delta(p)|^2}\{&(|I_2p^2 + R_2p + C_2|^2 + r_{21}^2C_2^2)|I_1p + R_1|^2 \\ &+ (|I_1p^2 + R_1p + C_1|^2 + r_{12}^2C_1^2)|I_2p + R_2|^2\}.\end{aligned} \tag{87}$$

This expression must, by Eq. 83, be integrated within the limits 0, ∞. This is most conveniently done by the table of integrals mentioned in the preceding paragraph. The result is, except for some numerical factor that is omitted,

$$[\epsilon^2] = \frac{d(ar_1 + ra_1 + 2ac - 2rd) + (rc - a)c_1}{ad(rc - a) - r^2d^2}, \tag{88}$$

[7] See pp. 369–370 of the treatise by James et al., footnote 2.

where

$$r = \frac{R_1}{I_1} + \frac{R_2}{I_2}, \qquad c = \frac{C_1}{I_1} + \frac{C_2}{I_2} + \frac{R_1R_2}{I_1I_2}$$

$$a = \frac{R_2C_1 + R_1C_2}{I_1I_2}, \qquad d = (1 - r_{12}r_{21})\frac{C_1C_2}{I_1I_2}$$

$$r_1 = 2\left[\left(\frac{R_1}{I_1}\right)^2 + \left(\frac{R_2}{I_2}\right)^2 - \frac{C_1}{I_1} - \frac{C_2}{I_2}\right]$$

$$c_1 = (1 + r_{12}^2)\left(\frac{R_2C_1}{I_1I_2}\right)^2 + (1 + r_{21}^2)\left(\frac{R_1C_2}{I_1I_2}\right)^2$$

$$a_1 = (1 + r_{12}^2)\left(\frac{C_1}{I_1}\right)^2 + (1 + r_{21}^2)\left(\frac{C_2}{I_2}\right)^2 + 2\left(\frac{R_1R_2}{I_1I_2}\right)^2 - 2\left(\frac{R_1}{I_1}\right)^2\frac{C_2}{I_2} - 2\left(\frac{R_2}{I_2}\right)^2\frac{C_1}{I_1}.$$

The minimization of error (88) depends on which of the parameters are fixed and which are variable. A great many cases could be con-

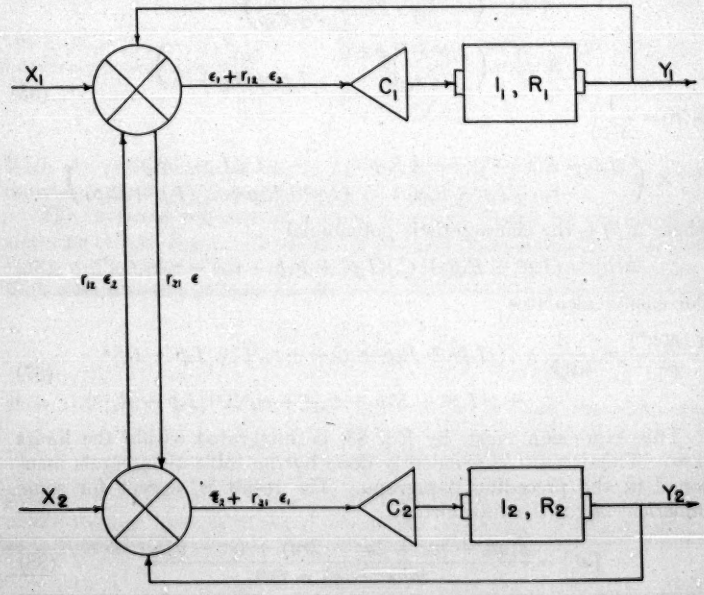

FIG. 4.

sidered. This is not undertaken here, and only a few simple remarks concerning the minimization will be made.

If R_1/I_1, R_2/I_2, C_1/I_1, C_2/I_2, and $r_{12}\cdot r_{21}$ are fixed, then $[\epsilon^2]$ is a minimum provided

$$\left|\frac{r_{12}}{r_{21}}\right| = \frac{(C_2/I_2)}{(C_1/I_1)}\sqrt{\frac{rd + (rc - a)(R_1/I_1)^2}{rd + (rc - a)(R_2/I_2)^2}}. \tag{89}$$

If the servo system is further idealized by assuming $I_1 = I_2 = 0$, then (88) is replaced by

$$[\epsilon^2] = \frac{(k_1 + k_2)^2 + (r_{12}k_1 - r_{21}k_2)^2}{k_1k_2(k_1 + k_2)(1 - r_{12}r_{21})}, \tag{90}$$

where

$$k_1 = \frac{C_1}{R_1}, \qquad k_2 = \frac{C_2}{R_2}.$$

If then k_1, k_2, and $r_{12}\cdot r_{21}$ are fixed the error is a minimum provided

$$\left|\frac{r_{12}}{r_{21}}\right| = \frac{k_2}{k_1}. \tag{91}$$

New Glass Is Precision-Machined by Chemical Process (*Electrical Engineering*, Vol. 70, No. 11).—A new kind of glass which can be precision-machined through the use of ultraviolet light, heat, and hydrofluoric acid to form intricately cut patterns of any desired shape and depth has been developed by the Corning Glass Works, Corning, N. Y.

This new glass is the latest in a series of photosensitive compositions first announced by Corning in 1947 to produce permanent 3-dimensional photographs in glass in a variety of colors. To the new composition, which is a special kind of photosensitive opal glass, an equally new process of chemical machining has been adopted, which produces, without the aid of mechanical tools, lace-like patterns hitherto considered impossible in glass.

The first step in the process is the printing of a design in the glass, using an ordinary photographic negative and ultraviolet light. Development then is accomplished by heating the glass to 1200 degrees Fahrenheit for the required length of time, usually about two hours. At this stage, a milk-white image appears in the otherwise transparent glass.

Next, the glass is immersed in a solution of hydrofluoric acid until the white areas are eaten through and removed, leaving the remaining unexposed glass in the exact form of the original pattern.

By varying the length and intensity of light exposure through the photographic negative, the depth of acid penetration in the glass can be accurately controlled from shallow etching to complete erosion. In this way, sculptured figures or contoured shapes can be made by using a continuous tone negative with proper degrees of shading.

Since chemical machining involves no mechanical stress, patterns, otherwise too complex to reproduce in glass or any other material without long and tedious skilled craftsmen, can be reproduced simultaneously in any number and with photographic accuracy.

Chemical machining is especially suited to perforating holes of any shape having diameters of only a few thousandths of an inch and numbering up to several thousands per square inch. Designs as irregular as lacework can be photoetched as readily as simple straight-line patterns.

The versatility of the process has already been demonstrated in the manufacture of printed electric circuits for electronic instruments. By using a photographic negative, in a single operation, a sheet of glass is cut into multiple pieces, each of proper shape and size, and each having identically etched circuit patterns and holes for fastening to a chassis. The pattern thus photoengraved in each piece of glass is filled with conducting metals to form an electric circuit of high precision and durability.

A rapid cure for malaria may be provided by primaquine, a new drug now undergoing extensive field testing by the Army. First tested on volunteer prisoners at the Statesville Penitentary, Joliet, Illinois, in cooperation with the University of Chicago, and later in Nicaragua and at Forts Benning and Knox, the drug gave a high percentage of cures among malaria cases with no subsequent relapse. Several hundred men returning from Korea will be treated with primaquine, while enroute to the U. S.; if this trial is successful it may be possible to cure malaria which has been suppressed by chloroquine, now used in Korea, without an individual knowing he had contracted the disease.

RESISTANCE OF PLATES WITH SMALL ELECTRODES *

BY

CHARLES S. McCLESKEY, JR.[1]

ABSTRACT

Using the image method, and with the aid of mathematics developed in the theory of crystal structure, it is possible to write formulae for the electrical resistance of any parallel plate. This has been done for the case of two hemispherical electrodes on one face of an infinite plate, with correction terms for small edge effects. The results have been applied to the metal thickness meter and confirmed by experiment.

The resistance of a uniform rectangular plate, measured between relatively small surface electrodes, is a problem of practical importance which may be solved by the method of images. It is evidently closely related to the well known (and exhaustively treated) geophysical problem of electrical conduction in stratified media; however, the simple series expressions obtained for the interelectrode resistance do not converge for a layer bounded on both sides by an insulating medium (1).[2] An improvement may be obtained by the rapidly convergent series obtained by Madelung (2) for the potential due to a charge lattice.

THE INFINITE PLATE

If the current problem is replaced by an equivalent electrostatic problem, the image system for the case of two electrodes upon an infinite plate is that shown in Fig. 1. Two charges occupy positions corresponding to the locations of the electrodes δ cm. apart, each generating an infinite row of images of the same sign at intervals of d cm. The "plate," between the surfaces S_1 and S_2 which are not crossed by flow lines, is $d/2$ cm. thick.

The potential ϕ due to a single line of charges (Q) at intervals d cm. is, from the paper by Madelung (2):

$$\phi = \frac{4Q}{d} \sum_{k=1}^{\infty} K_0\left(\frac{2\pi k r}{d}\right) \cos\frac{2\pi k x}{d} + \frac{2Q}{d} \ln\frac{2d}{r} + K_1. \qquad (1)$$

Here the field point is specified in cylindrical coordinates x and r, whose axis is the line of charges. The function K_0 is related to the standard

* This paper is an abstract of a Master's thesis, prepared at Louisiana State University, Baton Rouge, La.

[1] Deceased; formerly, Defense Research Laboratory, Austin, Texas.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

Hanckel function of the first kind by the equation:

$$K_0(x) = \frac{i\pi}{2} H_0^{(1)}(ix),$$

and K_1 is a constant.

Now the electrodes (supposed to be of infinite conductivity) may be given the shape of any equipotential surfaces. If the electrodes are sufficiently small these surfaces will be hemispheres, say of radius a. Then the following expression for their potential difference (V) is obtained:

$$V = \frac{4Q}{d}\left\{2\sum_{k=1}^{\infty}\left[K_0\left(\frac{2\pi ka}{d}\right) - K_0\left(\frac{2\pi k\delta}{d}\right)\right] + \ln\frac{\delta}{a}\right\}. \qquad (2)$$

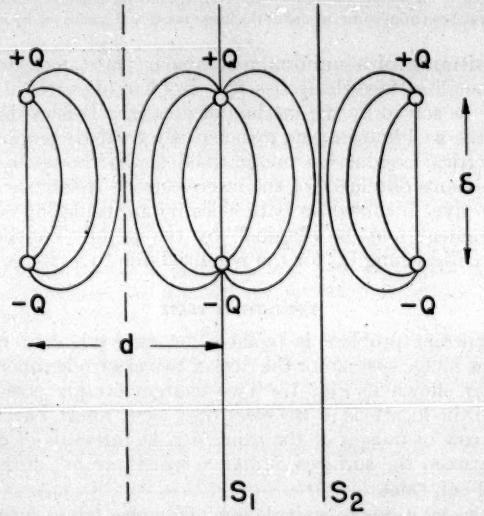

FIG. 1. The diagram shows the arrangement of the charges for an infinite plate which occupies the space between the two planes S_1 and S_2, perpendicular to the page.

Then applying the theorem relating resistance (R) and capacitance (C) in an extended medium (of specific resistivity ρ), one may write an expression for $\pi R/2\rho$, a quantity convenient for calculation. (In so doing it should be noticed that only half of the flux from Q lies within the "plate.")

$$\frac{\pi R}{2\rho} = \frac{1}{8C} = \frac{V}{4Q} = \frac{1}{d}\left\{2\sum_{k=1}^{\infty}\left[K_0\left(\frac{2\pi ka}{d}\right) - K_0\left(\frac{2\pi k\delta}{d}\right)\right] + \ln\frac{\delta}{a}\right\}. \qquad (3)$$

This expression is rapidly convergent when the plate is not too thick. When $d \gg \delta$, it is necessary to use another of the Madelung formulae which converges the more rapidly as d increases:

$$\phi = Q\left\{\frac{1}{\sqrt{x^2+r^2}} + \frac{1}{\sqrt{(d-x)^2+r^2}} - \frac{1}{d}\left[\psi\left(\frac{x}{d}+1\right) + \psi\left(2-\frac{x}{d}\right) - \frac{r^2}{4d^2}(\psi'')\right]\right\} + K. \tag{4}$$

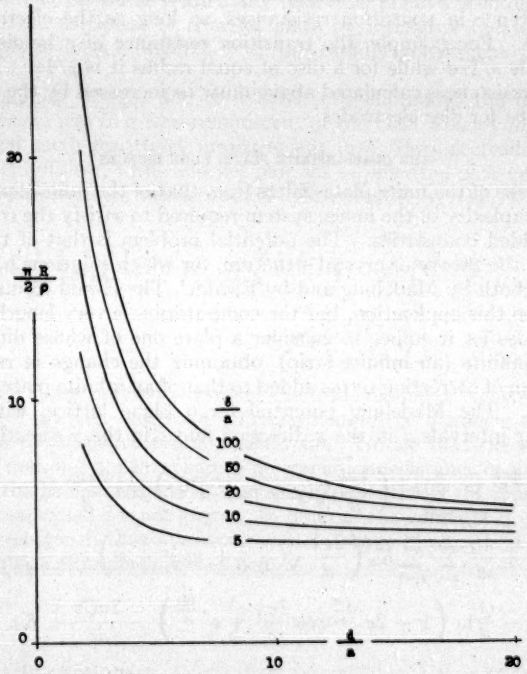

FIG. 2. The theoretical curves show the dependence of the reduced resistance $\pi R/2\rho$ on the relative thickness d/a for various values of the relative electrode distance δ/a.

Here ψ is the logarithmic derivative of the factorial function. When (4) is suitable, we have in place of (3):

$$\frac{\pi R}{2\rho} = \frac{1}{2}\left\{\frac{1}{a} - \frac{1}{\delta} + \frac{1}{\sqrt{d^2+a^2}} - \frac{1}{\sqrt{d^2-\delta^2}} + \frac{1}{d}\left[\frac{a^2-\delta^2}{4d^2}(\psi''(1) + \psi''(2)\cdots)\right]\right\}. \tag{5}$$

With Eqs. 3 and 5, the resistance for any combination of parameters may be found. Figure 2 illustrates the dependence of the resistance upon d and δ. Since a is assumed to be small, in a radially symmetric field, the resistance is inversely proportional to a. Numerical calculations show this over the range of values of a from very small to over half the plate thickness.

These results may be extended to a case of more than two electrodes by addition of Madelung potentials corresponding to the new image systems. Other electrode shapes may be accounted for by introducing the difference in transition resistances, so long as the electrodes are small (3). For example, the transition resistance of a hemisphere of radius a is $\rho/2\pi a$ while for a disc of equal radius it is $\rho/4a$. Then the reduced resistances calculated above must be increased by the constant $(\pi - 2)/8a$ for disc electrodes.

THE QUASI-INFINITE PLATE: EDGE EFFECTS

The case of the finite plate differs from that of the infinite plate only in the complexity of the image system required to satisfy the conditions at the added boundaries. The potential problem is that of the space lattice in the theory of crystal structure, for which solutions have been obtained both by Madelung and by Ewald. The Ewald formula (4) is suitable to this application, but the computation is very lengthy. For our purpose let it suffice to consider a plate one of whose dimensions remains infinite (an infinite strip), obtaining the change in resistance in the form of correction terms added to that of an infinite plate of equal thickness. The Madelung potential for a plane lattice, with equal charges at intervals d in the x direction and c in the y direction is

$$\begin{aligned}\phi = {} & \frac{4Q}{d}\sum_{k=1}^{\infty}\sum_{m=0}^{\infty} K_0\left(\frac{2\pi k}{d}\sqrt{(y+mc)^2+z^2}\right)\cos\frac{2\pi kx}{d} \\ & + \frac{4Q}{d}\sum_{k=1}^{\infty}\sum_{m=1}^{\infty} K_0\left(\frac{2\pi k}{d}\sqrt{(-y+mc)^2+z^2}\right)\cos\frac{2\pi kx}{d} \\ & - \frac{Q}{d}\ln\left(1 - 2e^{-\frac{2\pi z}{c}}\cos\frac{2\pi y}{c} + e^{-\frac{4\pi z}{c}}\right) - \frac{2\pi Qz}{dc} + K_2.\end{aligned} \tag{6}$$

If two electrodes, δ cm. apart, are symmetrically located along the length (c cm.) of a plate b cm. wide, we obtain:

$$\left(\frac{\pi R}{2\rho}\right)_{b\to\infty} = \left(\frac{\pi R}{2\rho}\right)_{\substack{b\to\infty\\ c\to\infty}} - \frac{1}{d}\left[\ln\left|\frac{\delta}{a}\sin\frac{\pi a}{c}\cot\frac{\pi\delta}{c}\right|\right],$$

and (7)

$$\left(\frac{\pi R}{2\rho}\right)_{c\to\infty} = \left(\frac{\pi R}{2\rho}\right)_{\substack{b\to\infty\\ c\to\infty}} + \frac{1}{d}\left[\ln\left|\frac{b}{2\pi\delta}\left(1 - e^{-\frac{\pi\delta}{b}}\right)\right| + \frac{\pi\delta}{b}\right].$$

When both correction terms are small, they may be added to obtain the resistance of a finite plate. If the electrodes are displaced parallel to a boundary from the position of symmetry, the result requires only simple changes. Such correction terms are of use in assessing theoretically the error of metal thickness meters operating on small plates or near an edge, for example.

APPLICATION: THE METAL THICKNESS METER

Over a long period of years many instruments have been devised to determine the thickness of a metal plate by resistance measurement. Their analysis is not difficult with the rapidly convergent potential series.

One popular design, due to Warren (5), eliminates the effects of contact resistances in a way reminiscent of the Gish-Rooney method of geophysical earth resistivity measurement (6). Two electrodes establish the field of flow, while another pair act as potential pick-ups. The potential difference of the secondary electrodes is, by Ohm's law and Eq. 1:

$$V = \frac{\rho I_0}{\pi d}\left\{2\sum_{k=1}^{\infty}\left[K_0\left(\frac{2\pi k r_1}{d}\right) + K_0\left(\frac{2\pi k r_2}{d}\right) - K_0\left(\frac{2\pi k r_1'}{d}\right) - K_0\left(\frac{2\pi k r_2'}{d}\right)\right] + \ln\frac{r_1' r_2'}{r_1 r_2}\right\}. \quad (8)$$

Here r_1 and r_2 are the two secondary electrode distances measured from the positive primary electrode. Primes denote the distances as measured from the negative primary electrode. Notice that the electrode radius, a, does not appear; there is no transition resistance in the result.

Since the Hanckel function decreases exponentially, the infinite sum is quite small when d is not great. In practice the values of r_1, r_2, r_1', r_2' can be chosen such that over some desired range of operation the logarithm only is of importance. Then

$$V = \rho\frac{I_0}{d}\cdot(\text{constant}). \quad (9)$$

Evidently two modes of operation are possible: either I_0 or V may be maintained constant, after which d is very simply related to the other. It will ordinarily be desirable that V be constant, so that d is directly proportional to I_0.

THE EXPERIMENTS

Experiments were performed to test the formula for the resistance of an infinite plate with two electrodes. The measurements were made upon electrolytic layers, since metal plates offer many difficulties in connection with the lead resistance and contact resistance. The elec-

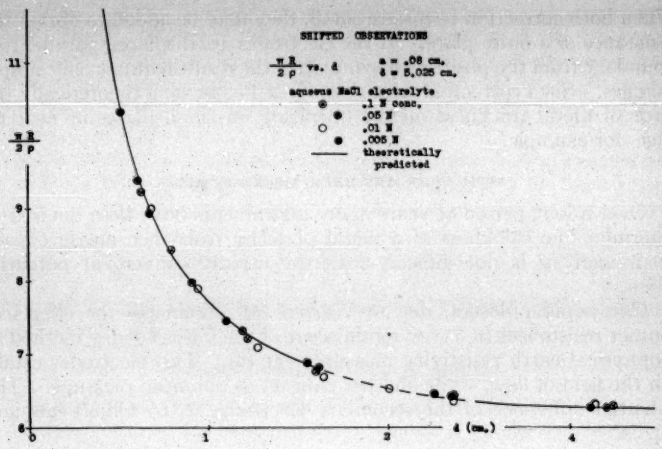

FIG. 3. The full drawn parts of the curve are calculated from Eqs. 3 and 5, the broken part is interpolated. The observational points, referring to various concentrations of NaCl solutions, are inserted after suitable shifts in the axes have been applied.

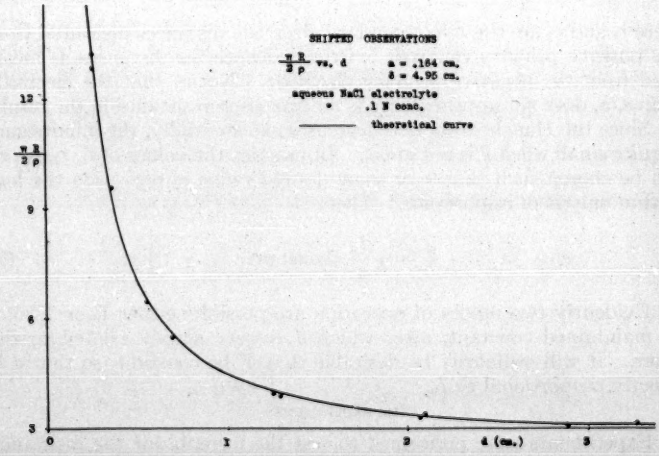

FIG. 4. Experimental results using larger electrodes are shown after the translation of axes. The solid line is theoretically obtained from Eqs. 3 and 5.

trodes were fixed in the bottom of a wax-lined pan, which could be filled to measured depths of liquid. An indentation around the bottom made it possible to achieve very small thicknesses before rupture of the layer by surface tension. As electrolyte, solutions of several salts at various concentrations were used.

To determine the resistance an ordinary bridge arrangement, with the customary capacity balance to sharpen the null, was used at a frequency of twenty kilocycles in order to reduce polarization. After the data for a particular solution were plotted as $\pi R/2\rho$ *versus* d, the axes were shifted to obtain a close fit to theoretical curves. The shift in thickness is due in part to error in the zero setting of the carriage of the traveling microscope used to measure depth (by focussing on cork dust upon the liquid surface). The difference in transition resistances of hemispherical electrodes (for which the curves were drawn) and disc electrodes (which were used) is absorbed in the resistance shift. However, both shifts are primarily dependent upon the chemistry of the solution, in a manner which has not yet been completely investigated.[3]

Numerous measurements were performed with two pairs of electrodes of different diameter. Solutions of NaCl, $NaNO_3$ and $Al(NO_3)_3$, in concentrations from 0.001 N to 0.5 N were used. In all cases the observations could be fitted in a satisfactory way to the theoretical curves after suitable shifts.

Figure 3 shows several sets of data for one pair of electrodes. Since the reduced resistance does not depend upon ρ, all such data fit one theoretical curve. Figure 4 shows the result of measurements upon the larger pair of electrodes.

Acknowledgment

The author is indebted to Dr. George Jaffé of Louisiana State University who has directed the investigation and guided the preparation of this report.

REFERENCES

(1) J. N. HUMMEL, *Am. Inst. Min. & Met. Eng.*, Tech. Publ. No. 418 (1931).
(2) E. MADELUNG, *Physik. Zeit.*, Vol. 19, p. 524 (1918);
L. V. KING, *Proc. Roy. Soc.*, Vol. A139 (1933), introduces a solution which reduces to that of Madelung, but no numerical calculations are reported.
(3) W. R. SMYTHE, "Static and Dynamic Electricity," New York, McGraw-Hill Book Co., Inc., second edition, 1950, § 6.15.
(4) P. P. EWALD, *Ann. d. Phys.*, Vol. 61, p. 253 (1921).
(5) A. G. WARREN, *J. I. E. E.*, Vol. 84, pp. 91–95 (1939). Numerical data and an empirical edge effects curve are included.
(6) A. S. EVE AND D. A. KEYS, "Applied Geophysics," second edition, Cambridge, 1933, p. 95.

[3] It has long been known that electrolytic solutions show an additional resistance upon application of alternating voltages. Compare M. Wien, *Wied. Ann.*, Vol. 58, p. 37 (1896) and C. Miller, *Phys. Rev.*, Vol. 22, p. 622 (1923).

NOTE ON "CORRECTION OF SPHERICAL ERROR OF A PENDULUM" *

BY

V. BUSH [1] AND J. E. JACKSON [2]

Since our article in the last issue went to press, we have found a decidedly interesting reference on the same subject, namely a posthumous article by Edouard Phillips in the *Comptes Rendus* of the French Academy of Sciences, Vol. 112, p. 178 (1891), which describes a scheme very similar to the one described in our article. It is strange that this method has been overlooked for so long.

December 17, 1951

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* Published in this JOURNAL, Vol. 252, No. 6, December 1951, p. 463.

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[2] Department of the Army, Washington, D. C.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS*

SUPPRESSION OF MICROWAVES BY ZONAL SCREENS

A method for alleviating one of the difficulties confronting the users of line-of-sight microwave communications—service interruptions resulting from ground-reflection effects—has recently been devised by the National Bureau of Standards through the application of optical methods and theories to microwave techniques. In particular, a method based on the classical Huygens-Fresnel diffraction theory has been developed by H. E. Bussey of the NBS staff for the suppression of the ground-reflected waves which are present in microwave radio relay link operations.

Interruptions in line-of-sight microwave communications may occur when the direct wave from the transmitter and the ground-reflected wave destructively interfere with each other at the receiver. Although it is possible to set the receiver at a point of constructive interference, subsequent atmospheric changes usually shift the spatial interference pattern (the so-called lobe pattern) of the source and its image sufficiently so that an interference minimum frequently occurs at the receiver.

In the NBS method reflected-wave suppression is achieved by setting a small screen of the proper size on the ground at the "reflection point" in the path. The reflected wave at the receiver is then substantially diminished, to an extent depending on the smoothness of the ground plane. The screen is designed to block only a small part of the reradiation from the ground to the receiver; the remainder of the reflected radiation adds up to zero at the receiver. The direct wave undergoes little or no modification during the suppression of the ground-reflected wave.

The NBS wave-suppression technique is based on the optical principle that the wave field transmitted from a point source to a point receiver under free-space conditions becomes zero if half of the first Fresnel zone is blocked so that the remaining diffracted contribution of the zone is halved in amplitude and unchanged in phase. The reduced contribution of the first zone is then cancelled by radiation from unblocked zones.

One of the screens developed by NBS takes the form of an opaque quarter-circle. The screen is erected on the ground at a point along the transmission path where the reflected ray from the transmitter strikes the ground. The plane of the screen is perpendicular to the path, and its radius is equal to that of the first Fresnel zone in this plane. Thus, the quarter-circle blocks one-half of the first Fresnel zone for the ground

* Communicated by the Director.

reflected wave; according to optical theory, the remaining contribution from this zone is cancelled and the effect of the image source is effectively eliminated at the receiver. There is also a large area surrounding the receiver position in which the effect of the image is nearly eliminated; consequently there will always be at least a partial suppression of the reflected wave at the receiver as meteorological fluctuations vary the position of the image source.

To obtain experimental confirmation of the reflected-wave suppression, the NBS investigators used a 4500-Mc. transmitter. Horizontally polarized signals were radiated from a 4- by 6-in. horn set about 14 ft. above the ground. The receiving antenna was an exact duplicate of the transmitting horn. The receiving antenna was located about 800 ft. from the transmitter and could be raised and lowered on a 50-ft. tower. The received power was recorded as a function of this variable height.

One of the NBS experimental wave suppressors is composed of two triangular screens with edges of 7.3, 6.6, and 5.4 ft. When the path was not obstructed by the suppressor, the receiver recorded a well-defined interference pattern of minimum and maximum signal strengths as the receiving antenna was raised and lowered on the tower. But when the triangular screens were placed in their proper position, the influence of the reflected wave was substantially eliminated at the receiver. The field strength of the remaining direct wave was 6 decibels less than that of the signal at a point of maximum interference when no suppressing screen was used. By moving the receiving antenna above and below its normal operating position in search of any nearby reflected-wave interference, it proved to be possible to determine whether the reflected wave was really suppressed or merely shifted in phase.

In practice, microwave radio relay paths are usually about 20 or 30 miles long, and the transmitters generally operate at a frequency of about 4000 Mc. (wavelength 7.5 cm.). Under these conditions, the first Fresnel zone is 80 to 100 ft. in radius at the middle of the path, and the radius of the main suppressed area at the receiving point is about 8 to 10 ft. Fortunately, the antennas commonly used for microwave relay transmissions are about 8 ft. in radius. To accomplish nearly complete suppression, the screen is positioned on the path to within a few feet of the center. Because the direct wave is diffracted, perfect suppression cannot be obtained in any instance, and some error in locating the screen can be tolerated. For a first zone of 80 ft., a satisfactory screen is a rectangular structure made of hardware cloth and mounted on poles 40 ft. high.

A number of NBS experiments using differently shaped screens have substantially confirmed the Bureau's application of optical theory to microwave techniques. They have indicated that troublesome ground reflections can be eliminated by small screens erected in the path, or

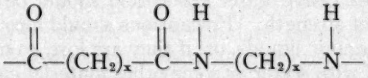
when technically feasible, by utilizing obstacles permanently located near the proper position in the path.

DURABILITY OF NYLON

An insight into the mechanism of the deterioration of nylons, also known as polyamides, has resulted from a recent extensive investigation[1] conducted by B. G. Achhammer, F. W. Reinhart, and G. M. Kline of the National Bureau of Standards. Thin films of the plastics were exposed to heat, ultraviolet radiation, and different types of atmospheres. The films were studied in various stages of degradation by a wide variety of techniques. It was thus possible to deduce some of the physical and chemical changes that had taken place. While the results obtained by NBS apply primarily to thin films of plastic-type nylon, they should provide a valuable basis for studies of the deterioration of other types of nylon in hosiery, textiles, brushes, and tires.

Improvement of the durability of plastic materials is one of the most urgent problems facing the plastics industry today. While the problem has been investigated extensively by means of accelerated tests involving one or more physical properties, empirical investigations of this type do not reveal the basic changes in the material and cannot be used to predict behavior in actual service. A complete understanding of the mechanism and kinetics of the fundamental reactions involved is needed. To provide such information, NBS is conducting a broad program of research on the degradation of plastics under the sponsorship of the Office of the Quartermaster General, Department of the Army.

The polyamides investigated by NBS were copolymers of nylon salts with ϵ-caprolactam, having the repeated structure.



Films 0.0001 to 0.0008 in. thick were exposed to heat, ultraviolet light, and various ambient atmospheres in apparatus specially designed to produce controlled degradation. They were also subjected to accelerated weathering and outdoor exposure tests.

In addition to analysis of the volatile degradation products with the mass spectrometer, both exposed and unexposed specimens were examined to obtain information concerning the changes in chemical and physical structure of the polymer. The techniques used included infrared and ultraviolet absorption, measurement of viscosities of solutions, measurement of dielectric constant and dissipation factor, photomicrography, X-ray and electron diffraction, electron microscopy, and treatment with

[1] For further details, see "Mechanism of the Degradation of Polyamides," by Bernard G. Achhammer, Frank W. Reinhart, and Gordon M. Kline, *J. Research NBS*, Vol. 46, p. 391 (1951) RP 2210.

organic liquids. Pyrolysis studies were made to study the breakdown of the long polymeric chain at high temperatures. Some physical properties were also determined on specimens in various stages of deterioration. While no single method was found to give a complete picture of the nylon degradation, the results from several of the techniques in combination provide a valuable insight into the mechanism of the process.

These results show that the general course of the degradation of the polyamides studied involves three different types of changes. First, the polymer molecules break at the C—N bond of the peptide group, creating smaller polymer molecules with the same unit of chemical structure. The fragments broken out of the chain are evolved as carbon dioxide, carbon monoxide, water, and hydrocarbons. Second, changes in molecular orientation or crystallinity may occur as a result of changes in hydrogen bridging, hydrocarbon (CH_2) packing, dipole rearrangement, and other secondary bonding among the polymer molecules. Third, the amount of strongly bound water and organic liquids such as ethanol may change. These materials, which act as plasticizers for the polyamides, are probably bound by hydrogen bridging to the oxygen molecule of the group. The changes in structure, molecular orientation, and chemically bound liquids produce marked changes in physical properties.

From the NBS investigation, it appears that the service life of polyamides could be extended by adding materials that would absorb ultraviolet radiation or by coating with a material that would keep the ultraviolet rays from the peptide group. Apparently it is unnecessary to protect against heat except under extraordinary circumstances.

Water and organic liquids act as volatile plasticizers. Thus, the water content of the plastic should be maintained at a relatively high value; however, excessive water absorption should be avoided because it results in loss of strength. Precautions should also be taken to prevent the loss of organic liquids, or, if they are lost, to make certain that they are replaced with water or some other material capable of forming a similar associated complex. Replacement of water and alcohols with less volatile water-insoluble materials bound to the polymer molecule in a similar fashion might provide a means of lengthening service life.

The detrimental effects of changes in crystallinity and orientation might be partially avoided by changing the chemical constitution of the polymer—for example, by alkyl-N-substitution. It appears, however, that as long as the carbonyl structure of the peptide group is present, a definite focal point of attack for degradation exists. If other groups, such as phenyl, were added, they might prevent deterioration by acting as radiant energy absorbers. However, they might also be capable of transferring their absorbed energy to the weaker C—N bond, causing it to break. Even if this happened, the resulting smaller polymer molecules would be prevented from crystallizing by the presence of the large side groups, and degradation would still be arrested to some extent. In

view of the present limited knowledge of such effects, it is not possible to predict how much of the problem can be solved in this way.

DETERMINATION OF MAGNESIUM IN CAST IRON

Addition of small amounts of magnesium to cast iron has received increasing attention recently because it imparts desirable ductile qualities to the iron, apparently by producing a nodular or spheriodal graphite structure. Because a variable part of the added magnesium is lost during the production of the cast iron, the final magnesium content must be determined by chemical or spectrographic analysis. Although spectrographic methods are quicker for routine examination, a suitable chemical method is needed, both for checking spectrographic standards and for use in laboratories not equipped for spectrochemical methods. An answer to this need is offered by a chemical method developed recently by John L. Hague and James I. Shultz of the National Bureau of Standards.

In this NBS method, the cast iron sample is dissolved in hydrochloric acid and the iron removed, after oxidation, as the chloride by ether extraction in a continuous extractor. Magnesium, together with part of the manganese, is then precipitated as phosphate from an ammoniacal citrate solution. After dissolving this phosphate precipitate in diluted acid, the manganese is removed by precipitation with ammonium persulfate as manganese dioxide. The magnesium is then precipitated as the phosphate, ignited, and weighed as $Mg_2P_2O_7$. Finally, the weighed pyrophosphate precipitate is examined for calcium by the sulfate-ethanol procedure, and any necessary weight correction is made.

The reliability of the method was checked by making a number of determinations on several NBS standard samples of iron to which had been added known quantities of magnesium. Between 0.01 and 0.10 per cent magnesium was added, the usual range in nodular cast iron. Calcium and cerium were also added to a few samples, as calcium may be present in some cast irons, and as cerium is sometimes added to induce a nodular structure. Results indicate that the NBS method of determination of magnesium in cast iron is applicable to most plain or low alloy irons, and also that moderate amounts of calcium and cerium do not interfere. Accuracy of the order of 0.002 per cent of magnesium was indicated for the range investigated.

NOTE: For details of the procedure, see "Determination of Magnesium in Cast Iron," by John L. Hague and James I. Shultz, *The Foundry*, Vol. 78, No. 10, p. 92 (October 1950).

THE FRANKLIN INSTITUTE

MINUTES OF THE STATED MEETING

December 19, 1951

The Stated Monthly Meeting of The Franklin Institute was held on December 19, 1951, in the Lecture Hall of The Franklin Institute. The President called the meeting to order at 8:15 P.M. Approximately 130 persons attended.

Mr. Nalle reported that the Minutes of the Stated Meeting for November were printed in full in the December issue of the Journal and, there being no corrections or additions, they were approved as printed.

The Secretary reported that according to the By-Laws, Article IV, Section 5, "Nominations for Managers shall be made in writing at the Stated Meeting in the month of December. Each nomination paper must be signed by at least two members, who shall certify that the candidate will serve if elected. After the nominations are closed, the President shall appoint three members, who are neither officers nor nominees, to act as tellers of election." He therefore read the names of the gentlemen whose nominations had been made in conformity with these By-Law provisions. They are as follows:

Henry B. Bryans	W. H. Fulweiler
Francis J. Chesterman	Wilfred D. Gillen
James Creese	Marshall S. Morgan
Rupen Eksergian	Edward Warwick

The President called for nominations from the floor, and there being none, the nominations were closed. To be tellers of election, Dr. George S. Crampton, Dr. Joseph Hepburn, and Mr. Penrose Hoopes were appointed.

Dr. Allen reported the following elections to membership as having been recorded for the month of November:

Active	42
Associate	29
Student	12
	83

The total Institute membership as of November 30 was 6,209.

The names of those who have been members of the Institute for twenty-five years and who will receive Silver Cards of Membership were read. These are listed under the Membership Section of this JOURNAL.

The annual James Mapes Dodge Lectures this season will be given on the afternoons of January 8 and 9. Mr. Watson Davis, Director of Science Service of Washington, D. C., will present "From Now On."

President Nalle introduced Mr. Hensleigh C. Wedgwood, President of Josiah Wedgwood & Sons, Inc., New York, who spoke on the subject "Josiah Wedgwood, F.R.S." This talk described how Wedgwood brought about his own Industrial Revolution in the field of ceramics, and in so doing, became the friend of most of the leading 18th century industrialists, scientists, artists and statesmen.

Following this interesting lecture, the meeting was adjourned with a rising vote of thanks to Mr. Wedgwood.

HENRY B. ALLEN,
Secretary.

MEMBERSHIP

ACTIVE MEMBERS ELECTED AT THE MEETING OF THE BOARD OF MANAGERS

December 19, 1951

ACTIVE FAMILY

Henry R. Eitzen
Louis Forman
G. D. Houtman
Paul E. Machemer
D. H. Renfrew
Col. James Truscott

ACTIVE

Jack R. Aldrich
James A. Britton
Alan W. J. Burns
W. W. Davis
Walter H. Eagan
Edward G. Fox
Charles S. Fruitman
Minor Harvey
Irwin Haupt
Randall Groves Hay
R. Edward Kay, Jr.
Mrs. Francis A. Lewis
C. J. McGonigal
James J. McKeegan
James J. McNally
Rev. John S. O'Conor
John H. Osler
H. William Overman
David M. Podvey
Maxwell L. Stoughton
Roland G. Swenson
Rev. R. Dunham Taylor
Walter S. Trout
Harlan L. Tuthill
Harold L. Warner
Charles H. Wilson

ACTIVE NON-RESIDENT

Benjamin de Gorter
Arthur B. Eaton

NECROLOGY

Mrs. Ida M. Craver '48
Benjamin Franklin '12
Edward T. Grandlienard '42
John Francis McCloskey, M.D. '36
Norman Mellor '92
Frank H. Rogers '21

NEW GOLD CARD MEMBERS HONORED

On November 17, the Institute paid tribute to three men who have just completed 50 years of continuous membership: James M. Caird, Municipal Water Supply Engineer, Troy, N. Y.; Gano Dunn, President of the J. G. White Engineering Corp., New York City; and W. E. Moore, President of W. E. Moore & Company, Pittsburgh, Pa. At the dinner held in Franklin Hall, Mr. James H. Robins, direct descendant of the first Chairman of the Board of The Franklin Institute, spoke about the meaning of membership in the Institute, saying, in part:

"Franklin Institute has always had a large number of distinguished representative members, ever since its original group 127 years ago. By 1901, when our honor guests joined, there were 2486 such members. As of this morning, the number had grown to over 6000—the size of an army brigade. I checked to see how we compared with similar organizations, and I couldn't find any institution with anywhere near this number of members in the United States—New York, Boston, Washington, Chicago or elsewhere. To exceed it, I had to think in terms of the National Geographic Society, which is more of a publishing house than a scientific institution like The Franklin Institute.

"The Franklin Institute offers many classes of membership to fit every purse, residential location and marital status. You might be surprised to learn how far-flung the membership is. We have members in 38 states and 10 countries.

"Some memberships are short-lived; but for the vast majority there exists a feeling of strong devotion and year-in and year-out loyalty to this fine institution and all it stands for. Membership ties are also shown by just such occurrences as tonight's, when three men who have been members for half a century have journeyed here—from northwest Pennsylvania, from upper New York State, and from New York City—to permit us to pay them our respects."

Mr. Robins then highlighted the many accomplishments of the three honored guests, by contrasting society of 50 years ago with that of today, in regard to pure water supply, electrification of industry, and construction engineering—the three fields of interest represented by the 50-year members.

Following Mr. Robins' introduction, Mr. Nalle, President of The Franklin Institute, presented gold keys and gold membership cards to the three men, who responded with the following remarks:

MR. CAIRD: "It may be of interest to learn how I became a member of The Franklin Institute. At the time I was special assistant professor of chemistry at the Rennselaer Polytechnic Institute.

"One morning the senior professor came in and advised that he had changed his suit and had left his money in the other suit and would I lend him some money. As he was the senior, and over me, I of course gave him the money. He then produced an application blank for membership in The Franklin Institute and told me to sign the same and if I did not get my money's worth he would repay me.

"I have been getting my money's worth ever since."

MR. DUNN: "It is not easy to express the sense of honor which I feel in receiving a gold key from The Franklin Institute and the tribute of this dinner in association with Mr. Caird and Mr. Moore, marking fifty years of continuous membership in The Franklin Institute. The obligation is all the other way.

"As I look back over my professional career, I realize afresh as I have realized again and again, the debt I owe The Franklin Institute for a large part of my professional education which has come from the study of its JOURNAL, reporting from time to time some of the most critical and important developments in electrical and other fields of engineering. Also of the greatest value have been the discussions of the papers which have constituted, as one looks back upon them, a history of the engineering art. These papers have been particularly well presented and well edited and they have included many epoch-making contributions.

"In voicing my gratitude to The Franklin Institute, there comes to mind the cynical observation attributed to Voltaire that: 'Gratitude is a desire for future favors.' Even if this were true, there cannot be many more years of these favors for a man 81 years old as I am. On this account I cherish another definition of gratitude which particularly applies in this case: 'Gratitude is a memory of the heart.' In this sense I feel toward The Franklin Institute an inextinguishable debt."

MR. MOORE: "I feel deeply honored at being present for this very delightful gathering. I have profited greatly from my membership in The Franklin Institute and it has been the source of a large part of my technical education.

"The carefully winnowed clippings published have always been of very great value. In one case the information from the JOURNAL gave me a complete defense in a rather complicated patent suit.

"I can't fully express my gratitude to the officers and officials for the courtesies shown."

The following are Gold Card members, having belonged continuously for 50 years or more. The dates are years of election to membership.

Arthur W. Howe '84
Alexander Sellers '84
William R. Armstrong '85
William Henry Bower '85
G. H. Clamer '91
Pierre S. duPont '92
Walter T. Lee '92
J. H. Granbery '95
Edward Woolman '95
Alan Wood, III '96
Morris E. Leeds '97
T. C. McBride '97
D. S. Jacobus '99
John J. McVey '99
E. A. Muller '99
C. C. Tutwiler '99
Frederic A. Delano '00
James M. Caird '01
Gano Dunn '01
W. E. Moore '01

The following are Silver Card members, having belonged continuously for 25 years or more. The dates are years of election to membership.

1902
Kern Dodge
John P. B. Sinkler

1903
Charles H. Howson

1904
Frank S. Busser
Elliott Curtiss
C. Mahlon Kline
George Woodward

1905
George R. Hall

1906
John L. Cox
Francis J. Stokes

1907
Darthela Clark
Frank Shaw Clark
Walton Forstall

1908
Herbert Berliner
George W. Borton
W. M. C. Kimber
John B. Rumbough

1909
Theobald F. Clark
Walton Clark, Jr.
J. B. Klumpp
James McGowan, Jr.
George Missimer, Jr.
T. Edward Ross

1910
Richard B. Brown
Joseph S. Hepburn
Joseph W. Lippincott
Horace P. Liversidge
J. M. Weiss

1911
Seth B. Capp
E. M. Chance
W. H. Fulweiler
M. M. Price
Charles Penrose
Frederic Rosengarten

1912
James Barnes
John Hampton Barnes
Morris Llewellyn Cooke
Edwin Elliot
Herbert E. Ives
Walter Palmer
Julian S. Simsohn
Howard S. Worrell

1913
Edward Bartow
George L. Coppage
Irenee duPont
Howard N. Eavenson
S. B. Eckert
Ralph E. Flanders
N. E. Funk
J. Kearsley M. Harrison
Lionel F. Levy
E. Mallinckrodt, Jr.
Frank M. Masters
Franklin P. McConnell
William Maul Measey
W. Chattin Wetherill
D. Robert Yarnall

1914
Russell L. Brinton
George S. Crampton
Ernest L. Huff
K. G. Mackenzie
Marshall S. Morgan
J. Edward Patterson
George Wharton Pepper

1915
Raymond Foss Bacon
C. E. Bennett
Thomas D. Cope
John C. Cornelius, Jr.
Justice C. Cornelius
H. Jermain Creighton
Thomas W. Elkinton
Richard Howson
S. Leonard Kent, Jr.
E. F. Kingsbury
W. Wallace McKaig
Walter F. Rittman

1916
Henry B. Allen
James G. Detwiler
Zay Jeffries
Richard C. McCall
Thorsten Y. Olsen
Frank H. Sauer
B. E. Shackelford
H. B. Spencer
Arthur Synnestvedt
Leo Wallerstein
Charles N. Weyl

1917
A. D. Chambers
Francis J. Chesterman
Walter S. Crowell
P. T. Dashiell
William J. Fitzmaurice, Jr.
John D. Gill
Rolfe E. Glover, Jr.
Harold Goodwin, Jr.
Arthur W. Lowe
Frank S. MacGregor
Edward E. Marbaker
Francis F. Milne, Jr.
C. H. Quinn
Sylvan D. Rolle
Robert L. Wood
Henry Woodhouse

1918
Charles H. Colvin
Fred H. Colvin
Charles H. Masland, 2nd
Walter O. Snelling

1919
Charles E. Brinley
Henry Colvin, 2nd
E. A. Eckhardt
Edwin D. A. Frank
George W. Furness
Frank H. Griffin
Hiram S. Lukens
H. Conrad Meyer
J. Howard Pew
Edmund G. Robinson
L. K. Sillcox
W. Leigh Smith
William Tiddy
Lucien Yeomans

1920
Robert Cameron Colwell
Howard W. Elkinton
Francis W. Hartzel
Chester Lichtenberg

P. S. Lyon
Haviland H. Platt
Carl D. Pratt
Winthrop R. Wright

1921
Roland L. Andreau
Leonard T. Beale
J. Ed. Brewer
Frederic Palmer, Jr.
Howard Stoertz
James Stokley
Haakon Styri
Edward R. Weidlein
S. Weinberg

1922
C. Douglas Galloway
Joseph F. Greene
L. H. Kinnard
L. W. McKeehan

1923
C. S. Chrisman
William Dubilier
Henry Clay Gibson
John Tracy Lay
Howard S. Levy
William C. Melcher, Jr.
John F. Metten
Lewis F. Moody
Richard H. Ranger
Jessie A. Rodman
Edwin G. Sagebeer
Coleman Sellers, 3rd
H. Birchard Taylor
Francis B. Vogdes
Joseph J. Vogdes

1924
Peter Abrams
H. Carl Albrecht
Clement Starr Brinton
Karl K. Darrow
Lee Davidheiser
William G. Ellis
T. R. Harrison
Valentine Hiergesell
Karl F. Oerlein
Edward B. Patterson
Harold Pender
Nicholas G. Roosevelt
Alfred O. Tate
Walter C. Wagner
James Lloyd Weatherwax
Alexander Wilson, 3rd
William Zimmermann

1925
Joseph S. Bennett, 3rd
Jack G. Binswanger
William L. Brown, 3rd
Marion Eppley
John Graham Foley
J. V. Giesler
C. R. Kraus
J. Kenneth W. Macalpine
William Stanton Monroe
W. F. G. Swann

1926
Charles B. Bazzoni
Neils Bohr
O. E. Buckley
William Findlay Downs
Henry C. Evans
Edward L. Forstall
Lewis H. Hendrixson
Charles S. Leopold
Alfred L. Loomis
J. H. Manning
William Clarke Mason
Clement B. Newbold
O. M. Patton
Charles S. Redding
Dexter N. Shaw
Sydney L. Wright
Charles S. Wurts, Jr.

JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in the JOURNAL within the next few months:

MOON, PARRY: A Proposed Musical Notation. (With discussions by several eminent musicians.)

AYRE, ROBERT S.: Transient Vibration of Linear Multi-Degree-of-Freedom Systems by the Phase-Plane Method.

MANDEVILLE, C. E. AND E. SHAPIRO: Radiations from Radioactive Rhodium (105).

NORINDER, HARALD: Investigation of Resistance and Power in Experimental Lightning Discharges.

SMITH, J. J. AND P. L. ALGER: The Use of the Null-Unit Function in Generalized Integration.

SILBERSTEIN, LUDWIK: Generalities Concerning "Simple" and Mixed Photographic Emulsions as Manifested by Their Characteristic Curve.

SAIBEL, EDWARD: Buckling of Continuous Beams on Elastic Supports.

BLADEL, JEAN VAN: Field Expandability in Normal Modes for a Multilayered Rectangular or Circular Wave-Guide.

ZADEH, LOFTI A.: A General Theory of Signal Transmission Systems.

KU, Y. H.: Resume of Maxwell's and Kirchhoff's Rules for Network Analysis.

THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

A new research group—the Solid State Physics Group—has been established by The Franklin Institute Laboratories for Research and Development under the direction of Dr. Foster C. Nix. Effective October 22, its founding represents one more step taken by the Institute toward its ultimate goal of providing adequate laboratory facilities for all types of basic research.

According to Dr. Nicol H. Smith, Executive Director of the Laboratories, the work of the new group will deal with the physics of metals, semiconductors, ferromagnetism, and other branches of the physics of solids. The other groups of the Research Laboratories are Chemistry and Physics, Electrical Engineering and Mechanical Engineering.

Personnel for the new group have been selected for their highly specialized training and intensive experience. In addition to the usual laboratory equipment, specialized apparatus has been installed. This includes a large electromagnet capable of producing very uniform

FIG. 1. Electromagnet.

Fig. 2. Nitrogen absorption apparatus.

and strong magnetic fields, apparatus for the measurement of the surface areas of very finely divided materials, and equipment for measuring the electrical and magnetic properties of solids.

One room is being devoted solely to electrical and magnetic measurements. Another has been completely outfitted with laboratory benches, modern power and gas outlets, and contains the analytical laboratory as well as the physical measurements and preparations. The irradiation project occupies two other rooms.

In addition, a complete furnace room—575 sq. ft. in area—for the preparation of metallurgical samples has been installed in the main Institute building. The room will be set up to process and heat, as well as prepare, samples of almost any kind and under almost any conditions.

The X-ray diffraction and electron microscope instruments have been placed under the control of the Solid State Physics Group, although they are available for use on other projects. The X-ray diffraction unit is used for studies with cameras or the spectrometer to provide information concerning the structure and chemical composition of many substances, and the

state of strain and work history of metal specimens. It also provides information concerning the crystal structure and the determination of particle size. The electron microscope is useful in studying particle size and the surface structure of metallic and non-metallic specimens and catalytic materials at any desired magnification up to 200,000. It also is useful for studying the structure of fibers, plastics, polymers and other materials that can be divided into very thin samples.

The initial work of the Solid State Physics Group consists of four Government projects: magnetostriction, ferromagnetism, effects of radiation on solids, and thermocouple materials.

These added facilities of the new group now are available to industry as well as to the Government, according to Dr. Smith.

LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostate Service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from 9 A.M. until 5 P.M.; Wednesdays and Thursdays from 2 P.M. until 10 P.M.

RECENT ADDITIONS

AERODYNAMICS

U. S. NAVY DEPT. BUREAU OF ORDINANCE. Handbook of Supersonic Aerodynamics. Vol. 1 & 2. 1950.

VAETH, J. GORDON. 200 Miles Up. The Conquest of the Upper Air. 1951.

AGRICULTURE

FREAR, DONALD E. H. Agriculture Chemistry. Vol. 2. 1951.

U. S. DEPT. OF AGRICULTURE. BUREAU OF PLANT INDUSTRY, SOILS AND AGRICULTURAL ENGINEERING. SOIL SURVEY STAFF. Soil Survey Manual. 1951.

ASTRONOMY

HYNEK, JOSEPH ALLEN, Ed. Astrophysics. 1951.

BIBLIOGRAPHY

BIERENS DE HAAN, DAVID. Bibliographie Neerlandaise Historique-Scientifique. 1883.

MÜLLER, FELIX. Führer durch die Mathematische Literatur. 1909.

O'HARRA, B. M. Bibliography on Zinc Retorts and Condensers. 1925.

SOCIETY OF PUBLIC ANALYSTS AND OTHER ANALYTICAL CHEMISTS. THE STANDARD METHODS SUBCOMMITTEE, comp. Bibliography of Standard Tentative and Recommended or Recognized Methods of Analysis. 1951.

U. S. WORKS PROGRESS ADMINISTRATION FOR THE CITY OF NEW YORK. Annotated Bibliography on Incineration, Carbonization & Reduction of Garbage, Rubbish & Sewage Sludge. 1939.

WHEELER, HAROLD L. A Bibliography on the Roasting, Leaching, Smelting and Electrometallurgy of Zinc. 1925.

BIOLOGICAL CHEMISTRY

ZIRKLE, RAYMOND E., Ed. Biological Effects of External Beta Radiations. 1951.

CHEMISTRY AND CHEMICAL TECHNOLOGY

AMERICAN CHEMICAL SOCIETY. COMMITTEE ON ANALYTICAL REAGENTS. Reagent Chemicals. 1951.

BAHRATT, WILHELM. Physikalische Messungsmethoden. Ed. 2. 1915.

BIJVOET, J. M.; KOLKMEYER, N. H.; AND MAGGILLAVRY, CAROLINE H. X-ray Analysis of Crystals. 1951.

BROWNLEE, RAYMOND B. AND OTHERS. Laboratory Exercises to Accompany First Principles of Chemistry. c1908.
BURGER, ALFRED. Medicinal Chemistry. Vol. 1. 1951.
DAVIDSON, ALBERT W. AND VAN KLOOSTER, HENRY S. Laboratory Manual of Physical Chemistry. Ed. 2. 1931.
DYSON, GEORGE MALCOLM. A Short Guide to Chemical Literature. 1951.
FELSING, WILLIAM A. AND WATT, GEORGE W. General Chemistry. Ed. 3. 1951.
HOPPE, JOHANNES. Analytische Chemie. Teil 1 & 2. 1905.
KAUFFMANN, HUGO. Allgemeine und Physikalische Chemie. Teil 1. Ed. 2. 1920.
KENT, ANDREW, Ed. An Eighteenth Century Lectureship in Chemistry. 1950.
KLEIN, JOS. Chemie Organischer Teil. Ed. 4. 1916.
MOILLIET, JOHN LEWIS AND COLLIE, BENJAMIN. Surface Activity. 1951.
Phase Transformations in Solids. 1951.
STANNARD, J. ELLIS. Chemistry. c1927.
SUMNER, JAMES B. The Enzymes; Chemistry and Mechanisms of Actions. Vol. 2^{-1}. 1951.
WATT, GEORGE W. Laboratory Experiments in General Chemistry and Qualitative Analysis. Ed. 2. 1951.
WEDEKIND, EDGOR LEAN WALDEMAR OTTO. Kolloidchemie. 1925.

CIVIL ENGINEERING

CHELLIS, ROBERT D. Pile Foundations. 1951.

ELECTRICITY AND ELECTRICAL ENGINEERING

DROTSCHMANN, CURT. Bleiakkumulatoren. 1951.
KEISTER, WILLIAM; RITCHIE, ALISTAIR; AND WASHBURN, SETH H. The Design of Switching Circuits. 1951.
THOMAS, HORACE A. Theory and Design of Valve Oscillators for Radio and Other Frequencies. Ed. 2. 1951.

ENGINEERING

JAGGAR, T. A. Abrasion Hardness. 1950.
MARQUERRE, K. Neuere Festigkeitsprobleme des Ingenieurs. 1950.
SPIELVOGEL, S. W. Piping Stress Calculations Simplified. 1951.

GENERAL

CASEY, ROBERT SABERT AND PERRY, JAMES WHITNEY, Ed. Punched Cards. 1951.
FISHER, DOUGLAS A. Steel Serves the Nation, 1901–1951. 1951.
REEVE, BERNARD AND THOMAS, P. W. Scale Model Ships; Their Engines and Construction. 1951.

GEOLOGY

JAGGAR, T. A. Steam Blast Volcanic Eruptions. 1949.

MANUFACTURE

BOLZ, ROGER W. Production Processes. Vol. 2. 1951.
DUNSHEATH, PERCY, Ed. A Century of Technology 1851–1951. 1951.
LEVITT, BENJAMIN. Oil, Fat and Soap. 1951.

MATHEMATICS

CROLL, M. Kartefunde. Teil 1 & 2. 1912.
DWYER, PAUL SUMNER. Linear Computations. 1951.
HERLAND, LEO. Wörterbuch der Mathematischen Wissenschaften. Vol. 1. c1951.
KACMARZ, STEPHAN AND STEINHAUS, HUGO. Theorie der Orthogonalreihen. 1951.
MEYER, VICTOR AND JACOBSON, P. H. Lehrbuch der Organischen Chemie. Vol. 2, Part 3 & 4. Ed. 1 & 2. 1920–27.
PICARD, CHARLES EMILE. Leçons sur Quelques Types, Simples D'Equations. 1927.
POLYA, GEORGE AND SZEGÖ, GABOR. Isoperimetric Inequalities in Mathematical Physics. 1951.

U. S. BUREAU OF STANDARDS. Tables of the Exponential Function E^x. 1951.
WEITBRECHT, WILHELM. Ausgleichungsrechnungnach der Methode der Kleinsten Quadrate. Teil 1 & 2. Ed. 2. 1912–1920.
WILLERS, FR. A. Graphische Integration. 1920.
WYLIE, CLARENCE RAYMOND. Advanced Engineering Mathematics. 1951.
YOUDEN, WILLIAM JOHN. Statistical Methods for Chemists. 1951.

MECHANICAL ENGINEERING

BRÖDNER, ERNST. Zerspanung und Werkstoff. Ed. 2. 1950.
FLORY, S., MANUFACTURING CO. Improved Contractors; Mines and All Kinds of Hoisting Engines. c1912.
THYBOUREL, FRANÇOIS AND APPIER, JEAN. Recueil de Plusieurs Machines Militaires, et Feux Artificiels pour la Guerre, & Recreation. 1620.

METALLURGY

AMERICAN INSTITUTE OF MINING AND METALLURGICAL ENGINEERS. INSTITUTE OF METALS DIVISION. The Solidification of Metals and Alloys. 1951.
AMERICAN INSTITUTE OF MINING AND METALLURGICAL ENGINEERS. IRON AND STEEL DIVISION. PHYSICAL CHEMISTRY OF STEELMAKING COMMITTEE. Basic Open Hearth Steelmaking. Ed. 2. 1951.
HEYN, E. AND BAUER, V. Metallographie. Teil 1 & 2. 1913.

METEOROLOGY

GEIGER, RUDOLF. The Climate near the Ground. 1950.

OPTICS

BENNETT, ALVA H. AND OTHERS. Phase Microscopy. 1951.
COSSLETT, VERNON ELLIS. Practical Electron Microscopy. 1951.
HINRICHS, W. Einführung in die Geometrische Optik. Ed. 2. 1924.

PATENTS

GREAT BRITAIN. COMMISSIONERS OF PATENTS. Catalogue of the Library of the Patent Office. Vol. 1 & 2. 1881–1883.
WOODCROFT, BENNET. Reference Index of Patents of Inventions, March 2, 1617 to October 1, 1852. 1855.

PHOTOGRAPHY

KESSLER, HEINRICH. Die Photographie. Ed. 5. 1916.

PHYSICS

BLASCHKE, WILHELM. Ebene Kinematik. 1938.
CHADWICK, JAMES AND ELLIS, CHARLES D. Radiations from Radioactive Substances. 1951.
DONNEEL, HEINRICH. Elektrochemie. Teil 1 & 2. Ed. 4. 1924–25.
FERMI, ENRICO. Elementary Particles. 1951.
FITCH, A. L. Fundamentals of Physics. 1929.
LEWIS, BERNARD AND ELBE, GUENTHER VON. Combustion, Flames and Explosions of Gases. 1951.
MAHLER, G. Physikalische Formelsammlung. Ed. 3. 1906.
PRAGER, WILLIAM AND HODGE, PHILLIP G. Theory of Perfectly Plastic Solids. 1951.
ROHMANN, HERMANN. Elektrische Schwingungen. Teil 1 & 2. 1914.
WILSON, WILLIAM. A Hundred Years of Physics. 1950.

TEXTILES

MASSOT, WILHELM. Textiltechnische Untersuchungsmethoden. Teil 1. 1924.
SILL, RALPH GUN HOY. Microbial Decomposition of Cellulose. 1951.

MUSEUM

The week of January which includes the day of Franklin's birth is observed as Printing Week, and affords opportunity for the nation to recall its indebtedness to that industry in which Benjamin Franklin labored. During the week of observation the name of Gutenberg will be generally and deservedly honored as that of the man who introduced movable type into Europe and started the printing industry as we know it today. While forbearing to deny Gutenberg the credit for an independent discovery, it should be pointed out that more than a quarter of a century earlier the Koreans were already printing books on an elaborate scale from movable cast metal.

In the year 1392 the Korean ruler, Tai Chong, known to Koreans as "The Father of Culture," had established a royal foundry for the casting of metal type in bronze. He had expressed dissatisfaction with the printing blocks then in use and decreed "that characters be formed in bronze and that everything without exception upon which I lay my hands be printed, in order to pass on the tradition of what these books contain." He declared that the beginning of what is now the basis of modern printing would be "a blessing to us to all eternity."

"In the second moon of the first year of Yung-lo" (1403) this enlightened ruler further ordained that the Korean laws and classics should be printed from this movable type for the benefit of those who might be called to the government of the country. He further declared that the cost of this undertaking would be defrayed from the royal purse, so that the expense would not fall upon his subjects.

The Museum is fortunate in the possession of two specimens of these early bronze type characters which antedate the printing of Gutenberg's Bible. One is the character "Ho" (harmony) dating from 1434, and the other is "T'ien" (Heaven) which dates from 1455. The two specimens are from the collection of Admiral Saito, who was Governor General of Korea and later premier of Japan. They were presented to the Museum by Dr. Judson Daland, who received them from the Admiral during a visit to Seoul.

Among the specimens of fine early printing on exhibition in the Graphic Arts section of the Museum are leaves from Joannes Gutenberg's Bible (1456); Nicholaus de Lyra's *Postillae super Bibliam*, printed in Rome in 1471 by Sweynham and Pannartz, who introduced printing into Italy; and J. de Voraigne's *Golden Legend*, printed at London in 1498 by Wynkyn de Worde, who was William Caxton's successor at the Westminister Press.

Visitors to the Museum will doubtless wish to see the colonial print shop with its furnishings similar to those used by Benjamin Franklin.

COMMITTEE ON SCIENCE AND THE ARTS

(Abstract of Proceedings of Stated Meeting held Wednesday, December 12, 1951.)

HALL OF THE COMMITTEE,
PHILADELPHIA, DECEMBER 12, 1951.

DR. RUPEN EKSERGIAN in the Chair.

The following reports were presented for final action:

No. 3232: Work of Edward C. Molina.

This report recommended the award of an Elliott Cresson Medal to Edward C. Molina, of East Orange, New Jersey, "For his important contributions to the mathematical theory of probability and for his significant contributions to the improvement of telephonic communications both by the application of mathematical probability to the study of telephone traffic and by the invention of switching equipment."

No. 3240: Spindle Nose Lathe Collet Chuck.

This report recommended the award of a Certificate of Merit to Arthur M. Stoner, of West Hartford, Connecticut, "In consideration of his achievement in producing outstanding advantages in the development of machine tool chucks resulting in an improved device known as the Spindle Nose Lathe Collet Chuck."

JOHN FRAZER,
Secretary to Committee

BOOK REVIEWS

AUTOMATIC FEEDBACK CONTROL, by William R. Ahrendt and John F. Taplin. 412 pages, diagrams, illustrations, 16 × 24 cm. New York, McGraw-Hill Book Company, Inc., 1951. Price, $7.50.

This is the first book, to our knowledge, to bring together the previously diverse technological fields of positioning servomechanisms, speed governors, pneumatically operated industrial controllers, and temperature regulators. This unification is greatly aided by the adoption by the authors, for the first time in any complete text, of the unified terminology (as well as the symbols) recommended recently by a joint committee of the engineering societies.

The theoretical treatment of the closed-loop control system, covered in the first six chapters, is based on the authors' previous text *Automatic Regulation* which was privately published in 1947 and which has been used since then as the text in courses given by Mr. Ahrendt at the University of Maryland. There is a profusion of graphs and charts, greatly clarifying the presentation. The essential mathematical treatment is given in a clear and simple form, starting with the setting up and Laplace transform solution of ordinary linear differential equations, then progressing to the more important complex variable and frequency spectrum methods of analysis.

The remaining seven chapters cover design considerations, nonlinearity and discontinuity, and examples in the industrial and military fields. A very comprehensive set of problems, with answers, suitable for either self study or teaching, is appended.

This text may well be considered seriously by the pedagogue as a teaching text, as it fills a long standing need for a unified treatment of the several fields in which automatic feedback control is important today.

B. B. YOUNG

TEXTBOOK OF ELECTROCHEMISTRY, by G. Kortüm and J. O'M. Bockris. 351 pages, diagrams, 17 × 25 cm. Houston, The Elsevier Press, 1951. Price, $7.00.

This book is a translation and revision of the first of two volumes of Kortüm's original work, *Lehrbuch der Electrochemie*. The authors have attempted to correct all of the errors and clarify many obscure points in the original German edition. This volume also sets forth the advances made outside of Germany during the years of World War II. Since the classical experiments and historical developments of electrochemistry are thoroughly covered in many source books, the authors have omitted much of this work from the text. Likewise, industrial processes and academic techniques are given only cursory treatment to keep the book from becoming too lengthy. The mathematics is not above the level of elementary differential and integral calculus so that the advanced undergraduate or first year graduate student can utilize the book without difficulty. Where some subjects are treated too briefly, bibliographies are included to give interested readers sources of more detailed information.

There are nine chapters in this first volume covering the following subjects: (*a*) the fundamental laws of electrochemistry; (*b*) the elements of thermodynamics; (*c*) the solvation of ions; (*d*) dissociation and activity coefficients for weak and strong electrolytes; (*e*) the theory of ionic attraction; (*f*) the determination of ionic mobilities and dissociation constants through conductance measurements; (*g*) electromotive force; (*h*) practical applications of potentiometric measurements; and (*i*) equilibria in solutions of weak electrolytes.

The authors' purpose is to set forth the fundamentals of electrochemistry in a concise and coherent manner without becoming too involved in specific details. They most certainly accomplish their purpose, and present a well-organized and readable book as well. A suggestion for improving the usefulness of this treatise would be to include an alphabetically ordered subject index at the end of the book. This first volume has been prepared excellently and forms an authoritative reference book and textbook of electrochemistry. The authors have focused attention on what has been done in this branch of science, and have tried to guide the thinking of both students and research workers in problems involving concepts of electrochemistry. No one active in the field of electrochemistry can afford to be without both volumes of this work.

DONALD H. RUSSELL

WHAT'S THE WORLD COMING TO?, by A. M. Low. 214 pages, 14 × 21 cm. Philadelphia, J. B. Lippincott Co., 1951. Price, $3.00.

People like to make predictions. They like to hear predictions. There is a certain fascination in peering into the future and there is a personal feeling of satisfaction in seeing these predictions fulfilled. And there is nothing that tickles the imagination more than proving the prophet wrong. However, it may be said that there is no future in prophecy, for in most cases when the prediction materializes the prophet has long since died.

What's the World Coming To? is a brilliantly written book by a master expositor. In a book of some 200 pages Prof. A. M. Low has lucidly portrayed the shape of things to come. Profound changes are due in the future and the author with a facile and rapid pen indicates the nature of the changes.

Professor Low goes from topics like atomic energy, robot electronic factories, electronic weapons, rockets, to foods of the future. In all these fields Prof. Low has a remarkable grasp of his subject and his intimate style is completely disarming. His blueprint of the future is one which can materialize because of his uncanny success as a prognosticator.

Some discrepancies have crept into his work such as the idea of increases in the world's cultivatible area provided by the melting of the polar ice. Actually the melting of the polar ice will raise the levels of the oceans by 150 feet and so inundate more land than is gained. Again, in a million years the sun will not lose 1 per cent of its mass—it will take thousands of millions of years for that. Dr. Low might have pointed out that already power installations using the heat from the earth's interior are in operation in Italy. These are not criticisms, for survey writing poses a most difficult problem.

What's the World Coming To? is an intriguing book and recommended for those who would glimpse the future, well remembering that "by attempting to look ahead we can hope to control the future in the widest sense."

I. M. LEVITT

SYNTHESIS OF ELECTRONIC COMPUTING AND CONTROL CIRCUITS, Vol. 27 of The Annals of the Computation Laboratory of Harvard University, 278 pages, diagrams, 20 × 27 cm. Cambridge, Harvard University Press, 1951. Price, $8.00.

One of the most popular and fast-growing sciences today is the branch of engineering known as electronics. Radar, television, and high speed computers are but a few of the recent outgrowths of work in this field. Though the fundamentals of the science are well-developed, many serious problems remain. The translation of a physical problem into block diagram form and the block diagram into standard electronic components are fairly straightforward, but the determination of the particular circuit design that uses the minimum number of components has remained a formidable, if not impossible, task. Volume 27 of the *Annals* offers an original solution to this problem.

A systematic algebraic method for determining the logical design of circuits is given. Ways are offered to prove that circuits which look quite different are sometimes equivalent. The methods suggested are illustrated by problems in the design of digital computing machinery, with special reference to components of the digital calculators developed at Harvard under the guidance of Howard Aitken.

Very little mathematical background is necessary to a full understanding of the material, and any engineer, electrical or otherwise, should be able to follow the gradual build-up of the method. The material presented is the outgrowth of years of experience during the teaching of a course of lectures entitled "Organization of Large-Scale Digital Calculating Machinery"; and a program of research in connection with electronic components for use in computing machinery, partly sponsored by the United States Air Force.

The idea of a switching function is used throughout. It refers to a binary function of binary variables (one the value of which is restricted to zero or unity), and is used to form equations that describe a triode, pentode, etc., by expressing the output as a function of the input. Following the discussion on single and multiple-output circuits, certain circuits depending upon feedback are treated. Components of digital computers are the subjects of the

concluding chapters. An appendix on the Switching Function of Four Variables concludes the text.

The text is double-spaced and well illustrated by functional diagrams, block diagrams, minimizing charts and tables of switching functions. A few references are given at the end of some of the chapters, and numerous examples are used to illustrate the work. The volume would make an excellent text for a course in this field, or a valuable addition to the library of any engineer working with control circuits or digital computation. Those who have used this book speak highly of it.

DONALD B. HOUGHTON

DIMENSIONAL ANALYSIS AND THEORY OF MODELS, by Henry L. Langhaar. 166 pages, illustrations, diagrams, 15 × 24 cm. New York, John Wiley & Sons, Inc., 1951. Price, $4.00.

The purpose of this well-organized little book is to present in a systematic manner the principles of dimensional analysis, to show its role in the theory of models, and to provide instructive examples of its applications in various fields of science and engineering. As distinct from the nature of some of the problems to which it has been applied, a thorough understanding of its essential features requires no more mathematical background than elementary algebra, including the theory of linear equations and a familiarity with the concepts of a "variable" and a "functional relationship" among variables.

The author, who has been giving a graduate course on the subject at the University of Illinois for several years, demonstrates this fact very convincingly in the first sixty pages of the book. A careful exposition of the concepts "dimension," "unit of measurement," "dimensional homogeneity" and "dimensionless product" is followed by a discussion of the process of formulation of a problem amenable to dimensional analysis, of the concept of a "complete set" of dimensionless products, and of the significance of Buckingham's theorem. Two chapters are devoted to the presentation of a systematic method of calculation of dimensionless products and of an algebraic theory of dimensional analysis culminating in a rigorous proof of Buckingham's theorem, both of which are original contributions of the author to the subject.

With the exception of the last chapter containing a brief exposition of the method of deriving dimensionless products from differential equations assumed to apply to the problem on hand, the remaining part of the book is devoted to applications of dimensional analysis in various fields.

The concepts of the theory of models, such as "similarity" (some writers prefer "similitude"!), "homology," "scale factor," "distorted model," are introduced in logical order and the practical considerations leading to the choice of a specific model are skillfully presented.

The use of dimensional analysis in problems lying in the fields of solid and fluid mechanics, electromagnetic theory, and the theory of heat, is illustrated by many well-chosen examples which are discussed in sufficient detail.

Each of the ten chapters contains worked-out examples and a large number of problems for the reader (the answers to which are listed at the end of the book), rendering it very suitable as a textbook.

This reviewer is confident that the present book will be welcomed by many as a congenial and intelligent guide into the realm of concepts of a theory which provides the tools for an efficient utilization of very incomplete "pre-existing knowledge" of a phenomenon in the layout of experiments designed to obtain additional information relating to it.

OTTO R. SPIES

HERBERT H. DOW, PIONEER IN CREATIVE CHEMISTRY, by Murray Campbell and Harrison Hatton. 168 pages, port., illustrations, 18 × 26 cm. New York, Appleton-Century-Crofts, Inc., 1951. Price, $3.50.

Herbert Dow was a pioneer in creative chemistry of an essentially American type. He was not merely a chemist interested in devising new and better chemical formulas; he was, in addition, an engineer who had the wisdom to perceive that if atoms were to be arranged there was a decided advantage in arranging them in carload lots. He recognized that in chemical manufacturing a piece of equipment could carry the same significance as the chemical formula.

That might not appear to be a special combination of qualities today, but it was unique in those days when young Dow conceived the idea of finding a use for all the brine that was running to waste in Michigan. It was not a time of conservation. Forests were being ruthlessly slashed, land was plowed so prodigally that much of it was rapidly converted into dust bowls, petroleum and natural gas were wasted as though the supplies were inexhaustible. Dow, at the age of twenty-three, was scandalized at the waste of brine and determined to do something about it. It took him seven years to do it and he had to learn some stern lessons, but he made the extraction of bromine from brine a profitable undertaking, and then proceeded to utilize the bromine in many ways. One of the principal lessons he learned was to have a profound respect for the industrial dollar, whether he received it or paid it. Another was that reasons for failure were not excuses for abandoning a process, but only incentives toward the removal of the obstacles. By dogged perseverance, Dow and his associates were able within a few years to develop some one hundred and fifty useful products.

As in so many instances where the growth of a company was largely due to the efforts of an individual who is in many respects a crusader, the company's history is marked with more of the enthusiasm and confusion of a crusade than the organized progress of a well-planned campaign. But Dow was always practical, and he became distinguished for the quantity and character of his ideas, as well as for his militant economy in manufacturing methods to hold down costs. It was largely through his determined efforts to implant research methods into an industrial chemical plant that a vast enterprise grew out of a modest bleach and bromine plant. The chemical industry is also under a debt to Herbert Dow for the courageous manner in which he strove to break the strangle hold that foreign cartels had placed upon the American market.

This book is not in any sense a complete biography. It is a description in non-technical language of how one man founded a chemical manufacturing company and played a major role in the creation of the American chemical industry. The authors have made good use of their material in preparing a readable account of the exertions of Herbert Dow, who was such an inspiring influence that, twenty years after his death, his spirit endures in the company he founded.

T. C.

TIME'S ARROW AND EVOLUTION, by Harold F. Blum. 222 pages, illustrations, 16 × 24 cm. Princeton, Princeton University Press, 1951. Price, $4.00.

A relationship between the second law of thermodynamics and organic evolution is a tempting and exciting possibility. In a remarkably readable and competent book Dr. Harold F. Blum, a member of the National Cancer Institute, shows how living structures may be treated as thermodynamic systems without violating the principle of irreversibility. He shows that no matter how carefully the energetics of living systems are examined, the above law holds.

In *Time's Arrow and Evolution* Dr. Blum has achieved a notable success in the arrangement and integration of material. After an introduction in which the basis for the book is established, there are three chapters dealing with the evolution of the earth. The analysis of necessary chemical and physical conditions for the beginning of life is then made and finally the origin of life is discussed, going from the spontaneous origin to the cosmic "seeding" of Arrhenius. Dr. Blum has set for himself a difficult task by entering into the domain of the chemist and the physicist, the astronomer and the geologist, the biologist and the geneticist and from these fields he has extracted the evidence upon which to base his theories. He has integrated these in a stimulating fluent style which is a tribute to his mastery of the subject.

There are some items which the astronomer might question. As an example, the evidence today seems to indicate a coeval beginning for the universe and the earth. This reviewer believes that Dr. Blum should have pointed out the similarity of materials in the Universe. As far as astronomers know today, our parent body, the sun, is a representative chunk of the Universe. Also, present day thinking indicates the existence of millions of solar systems in the universe and the conditions for life, similar to ours, just may be present in a great many of these solar systems. This brings up the existence of life on other worlds as an intriguing possi-

bility. Perhaps in another edition of his book he will include a discussion of this facet of the general problem of the origin of life.

I. M. LEVITT

SURFACE CHEMISTRY OF SOLIDS, by S. J. Gregg. 297 pages, illustrations, diagrams, 16 × 24 cm. New York, Reinhold Publishing Corp., 1951. Price, $8.50.

This book is a general survey of certain selected topics in the field of surface chemistry. These topics include: adsorption from the vapor or liquid phase on solids; determination of particle size and shape, and their influence on the bulk properties of powders and friction of metals; wetting and adhesion as influenced by adsorption; and catalysis, especially that part associated with chemisorption. The emphasis in this book is largely on adsorption and its effects, so that large areas of the field of surface chemistry are neglected, such as colloid phenomena, electrode effects, the kinetics of evaporation, etc. Actually the author points this out himself in the preface, and claims only an attempt to give a few of the topics in the field.

The author is essentially interested in describing quantitatively the phenomena that arise from adsorption. There is no intent to give a description of the fundamental physics of the processes taking place, nor yet of the detailed chemistry involved. It does have a very great virtue, however, in that it presents a picture of some very diverse fields based on the same fundamental process, that of adsorption, and succeeds in unifying these fields. The references given are not very voluminous, but they contain most of the basic references and texts in the fields covered. It must be mentioned, however, that even the fields selected by the author are not in every case thoroughly covered. For example, the discussion of interfacial tensions and the spreading of liquids over solids is quite sketchy and neglects large areas of work in this field, notably that of Bartell.

This book will be of most value to the newcomer in surface chemistry, as a review of the fields mentioned above, giving him a good picture of the phenomena involved before he goes on to more detailed texts.

A. D. FRANKLIN

BRITISH SCIENTISTS, by E. J. Holmyard. 88 pages, illustrations, 13 × 19 cm. New York, The Philosophical Library, 1951. Price, $2.75.

The purpose of this little book is to outline the contributions of those Britons who established many of the fundamental principles in science. It fulfills its purpose if the reader does not demand details. In a period when we are conscious of the clamorous claims of political nationalism it is pleasant to fall upon a small book which draws attention to the achievements of a nation's scientists without advancing any exaggerated claims. The distinguished editor of the international scientific journal *Endeavour* has drawn thumb-nail sketches of the work of some twenty-five British scientists in which he advances nothing but the most modest claims for his subjects. Roger Bacon, for example, is neatly trimmed down to size. On the other hand the author brings to light some half-forgotten accomplishments of those great men who are generally remembered for some single unsurpassed achievement. Although Michael Faraday receives treatment in fewer than six hundred words, we are reminded that, among other things, he elucidated the laws of electrolysis and discovered benzene. This little book will be useful to have at hand when the essential facts concerning the main British scientists are wanted.

T. C.

PHASE TRANSFORMATIONS IN SOLIDS, edited by R. Smoluchowski, J. E. Mayer and W. A. Weyl. 660 pages, illustrations, diagrams, 14 × 22 cm. New York, John Wiley & Sons, Inc., 1951. Price, $9.50.

In this volume is collected a series of papers by people working in the field of solid state physics. The papers were presented at a symposium at Cornell University in 1948, held under the auspices of the Committee on Solids of the National Research Council. Included among these papers are both reviews and the reporting of new results. Each paper is an individual

unit and can be read independently. The seventeen papers cover a great many aspects of the physics and other effects of phase transformations.

The subject matter falls into three main groups. The first few papers deal with theory. There are three papers on thermodynamics and the statistical mechanics of phase transformation by L. Tisza, by J. E. Mayer, and by J. G. Kirkwood. These papers report original work. F. Seitz has contributed a very complete review of the basic theory of diffusion of solids and its application to selected examples. R. Smoluchowski's paper performs the same function with nucleation of transformations in solids. M. Buerger discusses the structural changes involved. The second group of papers includes reviews on the study of transformations in non-metallic solids, with papers by O. K. Rice, M. L. Huggins, F. C. Kracek, J. F. Schairer, W. A. Weyl, and B. Matthias. Included are papers on argon, silver halides, silicates and glasses, and ferroelectrics. The third class of papers includes similar reviews on metals. The subject matter ranges through pure metals, order-disorder transformations in alloys, precipitation from solid solutions, and the eutectoid and martensite transformations in steel. The authors are C. S. Barrett, S. Siegel, A. H. Geisler, R. F. Mehl, A. Dube, and M. Cohen.

It is considered by this reviewer that this book is a very valuable contribution to solid state physics. Its value is indicated by the imposing list of contributors. It also benefits very much from inclusion of the discussions after the papers, in which are reported the widely diverse opinions that are currently held on these topics. As is inevitable with one volume in such a large field, the coverage is not complete. This is particularly true of the field of organic solids, which is completely unrepresented. However, those items that are included are given the most thorough treatment to be found anywhere in the literature. Special mention must be given to Dr. Geisler's article, which by itself is worth the price of the book. Dr. Geisler has not only completely surveyed the field of precipitation from solid solutions, but he has contributed a bibliography containing 830 references which he has cross-referenced in three or four different ways.

This book should be on the shelf of every solid state physics laboratory.

A. D. FRANKLIN

BOOK NOTES

THE LOST PHAROAHS, by Leonard Cottrell. 256 pages, plates, 15 × 22 cm. New York, Philosophical Library, 1951. Price, $6.00.

Inspired by the success of his Egyptology programs on B.B.C., the author offers this fascinating book, designed to interest the public in Egyptian archaeology to the extent of supporting further work with State funds. The book is well documented, accurate and written in a fast-moving style that keeps the reader in somewhat the same suspense that comes from reading "mystery" novels. The first chapter covers background history of Egypt, with an informative section on the development of engineering, astronomy, hieroglyphics, the calendar, and a system of state government. Succeeding chapters describe the work of the early archaeologists, the actual building of the pyramids and why they were built, tomb treasures and how they were discovered, the various types of burial chambers, and the detailed work involved in sorting, classifying and reconstructing the treasures from the tombs. The book is heartily recommended.

CAREERS IN PHYSICS, by Alpheus W. Smith. 271 pages, illustrations, 16 × 24 cm. Columbus, Long's College Book Co., 1951. No price.

Intended to give high school and college students an idea of what careers are possible in the field of physics, the book also is valuable to personnel managers responsible for hiring people in research laboratories. After outlining the broad aspects of physics as a profession and devoting two chapters to necessary educational preparation and the over-all picture of opportunities for physicists, the author briefly discusses specific careers such as teaching, power, industrial research, photography, instrumentation, communication, transportation, military

research, meteorology, etc. In each category, past accomplishments are outlined, as well as probable future needs. A useful book for career guidance.

MAN AND THE LIVING WORLD, by E. E. Stanford. Second edition, 863 pages, diagrams and illustrations, 15 × 22 cm. New York, The Macmillan Company, 1951. Price, $5.50.

Following the same general pattern of the first edition, which emphasized the human body and human interests in living things more than morphological and physiological matters, this second edition of Stanford's text for elementary biology courses incorporates the developments of the past 11 years. The book is designed as a lecture course, accompanied by demonstrations and visual aids, and, while it is lengthy, it may be cut down to suit the needs of most biology classes for students not planning to major in the subject.

ENCYLOPEDIA OF CHEMICAL REACTIONS, edited by C. A. Jacobson. Vol. 4, 790 pages, 16 × 24 cm. New York, Reinhold Publishing Co., 1951. Price, $14.00.

The fourth volume in this useful series covers the chemical reactions of iron, lanthanum, lead, lithium, magnesium, manganese, mercury, molybdenum and neodymium. Because many duplicates or near duplicates were encountered among these reactions, it was felt necessary to eliminate the duplications and substitute other reactions when possible. Difficulties arose over the fitting of certain of these substitutions into the alphabetical arrangement of the *Encyclopedia*, and so certain of the duplicates have been omitted. This volume will be a welcome addition to the first three in the series.

GENERATING STATIONS, by Alfred H. Lovell. Fourth edition, 431 pages, illustrations, diagrams, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1951. Price, $6.50.

The fourth edition of this standard textbook for students of power-plant design and associated transmissions incorporates the latest available economic procedures in investments, interests, depreciation, earning, taxes, etc. New sections have been added on the gas turbine as a prime mover, advanced designs in circuit breakers and relays, and generating-station auxiliaries. Also, new illustrations, problems and references bring the text up-to-date. The book offers a comprehensive picture of the economics of present day generating stations.

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NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

DR. ELLICE McDONALD, DIRECTOR

Seminar on The Chemistry of Nucleic Acids.—DR. M. STACEY of the University of Birmingham, England, addressed the Seminar of the Biochemical Research Foundation on Monday afternoon, September 17, 1951.

At Birmingham, the chemistry of the nucleic acids has been approached by Stacey and co-workers from the point of view of the sugar chemist. The two types of nucleic acid are simply polymeric forms of the purine and pyrimidine N-glycosides of D-ribose on the one hand and of 2-deoxy-D-ribose on the other joined through phosphate linkages. To form the necessary background, the syntheses and properties of a wide variety of deoxy-sugar derivatives have been studied.

The preparation of 2-deoxy-D-ribose has usually been carried out by the classical glycal method, but the best yields obtainable are 10 per cent or less. Other methods have been evolved and one improved method involves the reduction of a 2-tosyl derivative of D-arabinose by means of lithium aluminum hydride.

No striking difference in general physical properties was observed between corresponding derivatives of the normal sugars and the deoxy-sugars. Under some reaction conditions, e.g., oxidation and reduction, there is also little difference in chemical properties. The deoxy-sugars, however, are much less stable to acids, and their glycosidic derivatives are much more reactive in acidic hydrolysis. This is especially true of the furanose form of the deoxy-sugars, which is of special significance, since deoxyribose occurs in the nucleic acids in the furanose form.

It has also been demonstrated that deoxyribose constrained to a furanose form by substituent groups (as in the nucleic acids) tends to exist in the aldehydo-form when the reducing group is set free, to a much greater degree than do normal sugars. The introduction of additional deoxy-groups into the furanose ring further accentuates this tendency.

The reversion of deoxyribofuranose to the aldehydo-form is of importance in the Feulgen reaction for deoxyribosenucleic acid (DNA). Preliminary acid hydrolysis removes purine and pyrimidine glycosidic groups and occasionally a sugar phosphate group resulting in the formation of sufficient free aldehyde groups to react with the conventional aldehyde reagent.

The Dische test for deoxypentoses has been shown to depend on the conversion under acid conditions of the deoxy-sugar to ω-hydroxy-laevulinic aldehyde, which reacts with diphenylamine to give the character-

istic blue color. The reaction is given by the 2-deoxypentoses generally and is not specific for deoxyribose. The conversion of deoxypentoses to the laevulinic aldehyde derivative may also be involved in the Feulgen reaction, when sufficient acidic hydrolysis is used.

Evidence has been uncovered which indicates that about one in every two hundred deoxyribose residues in DNA holds a phosphate group at the aldehydic carbon rather than the usual N-glycosidic group. This phosphate linkage may lead to the formation of branched chains and may also be involved in strong hydrogen bonding with amino or hydroxyl groups of neighboring chains. This circumstance is bound up with the anomolous structural viscosity of undegraded DNA solutions for it is an unusually labile group. Cetyl-trimethylammonium bromide has been used as a differential precipitant for DNA and ribosenucleic acid (RNA).

It has been shown that in Gram-positive micro-organisms (e.g., *Cl. welchii*), the constituent absorbing the Gram stain is a nucleoprotein containing RNA. When Gram-positive bacteria are extracted with a bile salt solution, a magnesium salt of RNA is removed, and the residual cell structure is Gram-negative. The Gram-positive characteristic can then be restored by treating the extracted cells, the so-called cyto-skeletons, with the RNA in the presence of a reducing agent. DNA and simple nucleotides are not effective in restoring Gram-positive character to the extracted cells. The Gram-complex is established as being composed of the magnesium salt of a ribonucleoprotein, in which the basic protein contains at least 18 per cent of the total amino acids.

CURRENT TOPICS

Synchrotron for Cancer Research.—An atom-smasher that develops the most powerful X-ray beam ever generated specifically for the treatment of cancer is now being assembled at the University of California School of Medicine in San Francisco.

The machine is a synchrotron that produces an X-ray beam of 70 million volts. With it, University of California scientists will do some pioneering research on the possibility that ultra-high energy X-rays have some advantages in treating deep-seated cancer.

The synchrotron, purchased with funds from the Atomic Energy Commission, was recently delivered. It was designed and constructed at Schenectady, N. Y., by scientists and engineers of the General Electric Company's Research Laboratory and General Engineering Laboratory. It has operated successfully in tests at Schenectady.

The new cancer-fighting weapon is being reassembled in a two-story building constructed especially to house it and its associated laboratory. Total cost of the machine and the building, all financed by AEC funds, is about half a million dollars.

Dr. Robert S. Stone, professor of radiology and one of the pioneers in the application of nuclear physics in medicine, is director of the project.

Dr. Gail Adams, physicist in charge of the operation of the machine, came to the School of Medicine from the University of Illinois, where he played an important role in the development of the 20 million-electron-volt betatron, presently the most powerful X-ray weapon used in cancer treatment.

Dr. Adams explained that the new synchrotron will not be used in treatment of human patients for some time to come. The early work will be devoted to determining the effect of the ultra-high-energy X-rays on animals.

Dr. Adams explained that work with the 20 million-volt betatron indicates it is possible to put a higher dose of radiation in deep-seated tumors than with lower voltage machines. At 70 million volts this effect may be even more pronounced. The machine will determine whether this is the case.

The synchrotron will have considerable flexibility, since it can be adjusted between 20 and 70 million volts.

When assembled, the 16-ton machine will be mounted on huge trunnions, somewhat like an ordinary X-ray machine. By means of powerful auxiliary machinery, the magnet can be moved to adjust precisely the X-ray beam.

When in operation, the beam will be directed toward the side of the building which rests against Mt. Sutro. Since the beam is very sharp and penetrating, there is little danger from radiation in any other direction. However, the 3-ft. thick concrete walls of the synchrotron room provide extra precautionary protection.

The synchrotron operator and doctors performing treatment or experiments will be able to observe subjects through a glass tank filled with water. The water will provide extra protection to the observers.

In addition to the synchrotron, the new building will house necessary physical and chemical laboratories for studying the basic effects of the radiations, animal quarters for investigating the biological effects of the radiations, and rooms for examining patients.

Chloromycetin for Yaws and Tropical Ulcer.—Yaws and tropical ulcer, two of man's oldest and most disfiguring maladies, respond favorably to treatment by the antibiotic Chloromycetin, according to a recent study in Haiti.

Eight patients with serious cases of yaws were able to leave a hospital after one week's dosage with Chloromycetin, and all lesions were healed when checked six weeks later.

The study was conducted in Haiti by Dr. Eugene H. Payne of Parke, Davis & Company of Detroit, which supplies the antibiotic; Dr. Athemas Bellerive of the Haiti Department of Health and Dr. LaFont Jean of Fort Liberty, Haiti. Their findings were reported in the medical publication, *Antibiotics and Chemotherapy*.

Yaws, an infectious disease found in many tropical countries, has been known since ancient times and is believed by some medical men to have been "the leprosy" described by Moses in the Bible in the thirteenth chapter of the Book of Leviticus.

Tropical ulcer, found in the same surroundings, also was studied by the three investigators. Of twenty-one tropical ulcer patients treated with Chloromycetin, twelve were healed or almost healed after one week, three made no progress, and the remaining six disappeared after treatment and could not be observed.

New Line-Operated pH Indicator and Vacuum Tube Voltmeter.—New line-operated pH Indicator and Vacuum Tube Voltmeter developed by Leeds & Northrup, Philadelphia, is unaffected by normal fluctuations in line voltage or by zero drift of the amplifier. A converter-type instrument, its circuit employs d.c. to a.c. conversion, a.c. amplification, and conversion back to d.c. for voltage feedback. Conversion stabilizes zero; feedback stabilizes gain.

Used as a pH Indicator, the instrument reads directly in pH; makes glass electrode measurements to within ± 0.02 pH on grounded or ungrounded solutions. Continuous pH 0 to 14 scale eliminates range-changing. For manual lab use, temperature compensator is graduated from 0 to 100 C. in 2 degree steps. If the instrument is used as a continuous indicator on industrial process pH, automatic temperature compensation can be provided.

As a Vacuum Tube Voltmeter, the new indicator can be used with any high or low impedance electrode system which develops potentials within range of the instrument. External circuits can have up to 2000 megohms resistance. Current drain does not exceed 1×10^{-12} amp. at null balance. Four additional voltage scales of 0 to + 700, 0 to −700, 0 to + 1400, and 0 to −1400 millivolts are available for redox, potential, and polarized current measurements.

The instrument is built for hard use. Sturdy, modern case is made of grey-enameled aluminum. Step-by-step instructions for making all pH and millivolt measurements are inscribed on panel below meter.

Dynel Work Clothing.—Workers and miners, chosen to test an entirely new kind of work shirt and pants, are singing the praises of dynel, a remarkable new material so tough it can be soaked in acid without harm!

They report dynel's great resistance to chemicals will do away with frequent, expensive replacement of work clothes necessary in the past. Dynel, a soft but strong man-made fiber, developed by science, will have many uses in textiles, from men's socks to blankets, but its resistance to chemicals shows up most dramatically in work clothing.

In one western mine, acid water drippings and fumes ruin cotton shirts in a few weeks. Two dynel shirts have been worn in the mine on alternative days by one miner for over a year and still look good.

In a plant that processes aluminum chloride, zirconium and titanium tetachloride, workers often had to throw away their old-style cotton garments before a single eight-hour shift was over. They have now worn dynel shirts and pants for several months. Despite acid spatterings and frequent launderings, the dynel clothing does not even show appreciable signs of wear. How long the new shirts and pants will last cannot yet be predicted, since to date, as a filter press operator puts it, "They wear like iron."

Other types of work where dollar and cents savings have been made by using dynel clothing are electro plating, storage battery manufacture, plants producing or handling chemicals such as sulfuric acid, and oil-refineries.

The discovery of dynel means considerable savings to workers who, until now, have had to spend many dollars a year to replace work clothing. Dynel shirts and pants are union-made by Chem-Wear, Inc., New York.

The most startling characteristic of dynel is the fact that it can be boiled in many strong acid solutions without even shrinking. In certain petroleum cracking operations, for instance, the spillage of certain chemicals on work clothing can be removed only in sulfuric acid. Workers there have given a warm welcome to dynel as the only material which can take this rugged treatment without disintegrating. Workers with radioactive materials have found that dynel shirts and pants can be decontaminated in acid baths without damage, then washed and worn again without chancing exposure to retained radioactivity.

Despite its toughness, dynel work clothing is soft, comfortable and neat, looks like fine flannel. It wears wonderfully, is available in warm garments for winter and cool for summer—important in the varying temperatures found in many factories. Moths starve to death rather than eat it and it does not mildew. It is not a "treated" fabric; its acid resistance and other qualities are inherent and permanent, no matter how often it is washed.

Dynel work shirts and pants stand up excellently under frequent washings in the usually harsh soaps and detergents used on industrial garments. The fabric sheds dirt and grease, even wax, quickly and cleanly. Paint can be removed from it with such strong paint removers as are used on old wood, yet without harm to the fabric. It is not weakened by washing and does not shrink. It dries in a fraction of the usual time. If hung out to dry, it dries so free of wrinkles that it requires no ironing.

A petroleum plant machinist who has worn dynel shirts and pants for six months says: "This is the best work outfit I ever had. And it looks good enough to wear on Sunday!"

World Metallurgical Congress.—Approximately 500 foreign metallurgists, representing 37 nations, attended the recent World Metallurgical Congress held in Detroit. Sponsored by the American Society for Metals, the Congress was the first international conclave concerned with a world science of metals.

Molybdenum, titanium, boron and newly announced synthetic metals were the visible accomplishments of up-to-the-minute metallurgy as demonstrated in exhibits and lectures at the Congress: molybdenum was disclosed as now

ready for jet engine manufacture; boron was exhibited publicly for the first time (one ounce of boron added to one ton of steel cuts other less available ingredients nearly in half); and titanium may soon replace steel parts in planes and light steel construction, the experts predicted.

Frank X. Higgins, Project Administrator for the Economic European Cooperation office, said the Congress was the largest ECA technical assistance program ever staged and successfully brought about an information exchange between American and foreign metal men. He considered most important of all the personal contact the visitors had in living for a month and a half among our people, observing us in more than 60 cities and investigating the greater degree of cooperation we have between management and labor.

Dry Battery Powered Repeating Flash Unit.—A new method of taking flash pictures was described recently at the annual convention of the Photographic Society of America at Detroit by P. A. Marsal and N. M. Potter, Research Engineers with the National Carbon Company in Cleveland, Ohio.

The devices demonstrated use a new gas-filled flashtube in place of expendable flash bulbs, and are completely portable. They weigh only 6 to 8 lb. and are entirely self-powered by "Eveready" "B" batteries of the kind widely used in small portable radios.

Employing the "battery-capacitor" principle, the flash units have sufficient light output to make satisfactory daylight Kodachrome transparencies at a lamp-to-subject distance of ten feet, and an aperture setting of $f/3.5$. With Super-XX film, negatives of good density are obtained with an $f/11$ aperture at the same distance. The light output was said to be equivalent to that from a No. 5B flash lamp at a shutter speed of 1/100 second. The operating cost was claimed to be about one cent per flash, which is less than one-tenth of the cost when using expendable bulbs. One of the power sources shown was said to have capacity for a total of fifteen hundred flashes.

In operation, electrical energy is transferred from the "B" battery to electrical capacitors between flashes, where it is stored until a picture is taken. The minimum interval between flashes was shown to be as short as 3 or 4 seconds. At the instant of flashing, a high voltage trigger pulse causes the stored electricity to surge through a transparent U-shaped flashtube. The U-tube then emits a high intensity pulse of light that has an effective duration of about one millisecond. This short exposure time greatly reduces poor image definition caused by subject or camera movement—particularly important when miniature cameras are used. With correct synchronization all of the light pulse is accepted by the camera shutter, from slow speeds up to at least 1/400 second.

One of the repeating flash power sources demonstrated included a "slave" feature which triggered the flashtube on either light or sound pulses. This makes it possible to use one or more supplementary repeating flashlight sources, triggering them with the light source on the camera. Sound triggering of repeating flashlight sources, aside from its novelty interest, finds real use in the research laboratory in mechanical motion studies.